International Journal of Thermal Sciences 99 (2016) 258–270

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Estimation of thermal conductivity, heat transfer coefficient, and heat flux using a three dimensional inverse analysis

Farzad Mohebbi*, Mathieu Sellier

Department of Mechanical Engineering, The University of Canterbury, Private Bay 4800, Christchurch 8140, New Zealand

ARTICLE INFO

Article history:
Received 1 June 2015
Received in revised form
13 August 2015
Accepted 2 September 2015
Available online 1 October 2015

Keywords:
Inverse heat transfer
Elliptic grid generation
Sensitivity analysis
Finite difference method
Conjugate gradient method

ABSTRACT

This paper presents a numerical inverse analysis to estimate the thermal conductivity, the heat transfer coefficient, and the heat flux in three dimensional irregular bodies in steady state heat conduction problems. In this study, a 3-D elliptic grid generation technique is used to mesh the irregular body. The 3-D Laplace equation is solved in the computational domain to compute the temperature at any grid point in the meshed body. A novel and very efficient sensitivity analysis scheme is introduced to compute the sensitivity coefficients in gradient based optimization method. Using this sensitivity analysis scheme, one can solve the inverse problem without need to the solution of adjoint equation. The main advantages of the sensitivity analysis scheme are its simplicity, accuracy, and independency of the number of the direct problem solution from the number of the unknown variables which makes the numerical inverse analysis presented here very accurate and efficient. The conjugate gradient method (CGM) is used to minimize the objective function which is the difference between the computed temperature on part of the boundary and the measured temperature. The obtained results confirm that the proposed algorithm is very accurate, robust, and efficient.

1. Introduction

Over the past decades, the inverse analysis has been used in the heat transfer field for the determination of unknown quantifies such as heat flux, thermal conductivity, and heat transfer coefficient. In direct heat transfer problem, the boundary conditions, the thermo-physical properties, the geometrical configuration, and the heat flux are known and the heat transfer problem may be solved to determine the temperature distribution in the conducting body. In contrast, the inverse heat transfer problem (IHTP) is concerned with the determination of the boundary conditions, the thermo-physical properties, the geometrical configuration, and the heat flux from the knowledge of the temperature distribution within the conducting body. Mathematically, the inverse heat transfer problems are ill-posed. Ill-posed problems are inherently unstable and very sensitive to the inaccuracy in input data. Therefore, development of new computational schemes is necessary to overcome the numerical ill-posedness [1].

Inverse analysis has been used in the determination of thermo-physical properties such as the thermal conductivity and the heat transfer coefficient [2–14], the estimation of heat flux [15–20], and the estimation of the boundary shapes of bodies [21–25], to name a few. The 3-D inverse heat conduction problems have been treated much less than the 2-D problems due to the complexity of the problem and the high computational cost. As will be shown, however, using the 3-D numerical algorithm presented in this study, the unknown quantities may be estimated accurately in an integrated numerical framework. The two dimensional case using the same numerical procedure is successfully studied [26]. The objective of this study is to estimate the thermal conductivity, the heat transfer coefficient, and the heat flux in a 3-D irregular body both separately and simultaneously. In the proposed numerical algorithm, a 3-D elliptic grid generation technique is used to generate a mesh over the irregular body and solve for the heat conduction equation. The elliptic grid generation technique and the steady heat conduction equation both satisfy Laplace equation and therefore much of the effort put into programming the relations required to generate the grid over the physical domain can be reused to solve the heat transfer problem. The minimization of the objective function is performed using the conjugate gradient method which is a gradient based optimization method. A novel

* Corresponding author.
E-mail address: farzadmohebbi@yahoo.com (F. Mohebbi).

http://dx.doi.org/10.1016/j.ijthermalsci.2015.09.002

Nomenclature

$d^{(k)}$	direction of descent at iteration k
$\dot{q}$	heat flux (W/m^2)
h	heat transfer coefficient (W/m^2 °C)
Ja	Jacobian matrix
J	Jacobian of transformation
$\mathcal{J}$	objective function
k_T	thermal conductivity of the solid body (W/m °C)
n	outward drawn unit vector
T	temperature (°C)
T_m	measured outer surface temperature (°C)
T_∞	ambient temperature (°C)
x,y,z	Cartesian coordinates in the physical domain

Greek symbols

$\beta^{(k)}$	search step size at iteration k
Γ	boundary
$\gamma^{(k)}$	conjugation coefficient at iteration k
Ω	domain
ξ, η, ζ	Cartesian coordinates in the computational domain

Subscripts

i	grid index in ξ-direction
j	grid index in η-direction
k	grid index in ζ-direction
M	number of grid points in the ξ-direction
N	number of grid points in the η-direction
L	number of grid points in the ζ-direction

Superscript

k	iteration number

and very efficient sensitivity analysis scheme is introduced to solve the sensitivity problem without requiring to solve the adjoint problem. An explicit expression for the sensitivity coefficients is derived which enables us to compute the sensitivity coefficients in one single solve. The finite difference method is used to approximate the derivatives in the computational domain.

The three dimensional numerical algorithm presented in this study is sufficiently general by which we can determine the thermal conductivity, the heat transfer coefficient, and the heat flux applied on part of the boundary of a general three-dimensional region as long as the general three dimensional region can be mapped onto a regular (cuboid) computational domain. The method can be applied to any three dimensional body which can be meshed by the body-fitted grid generation technique (based on both PDE and algebraic methods).

2. Governing equation

The mathematical representation of the steady state heat conduction problem of interest here can be expressed as below (see Fig. 1)

$$\nabla^2 T = 0 \ \text{in physical domain} \ \Omega \tag{1}$$

subject to the boundary conditions

$$\frac{\partial T}{\partial n} = \frac{\dot{q}}{k_T} \ \text{on boundary surface} \ \Gamma_5 \tag{2}$$

$$\frac{\partial T}{\partial n} = -\frac{h_i}{k_T}\left(T_{\Gamma_i} - T_{\infty_i}\right) \ \text{on boundary surface} \ \Gamma_i, \ \ i = 1,2,3,4,6 \tag{3}$$

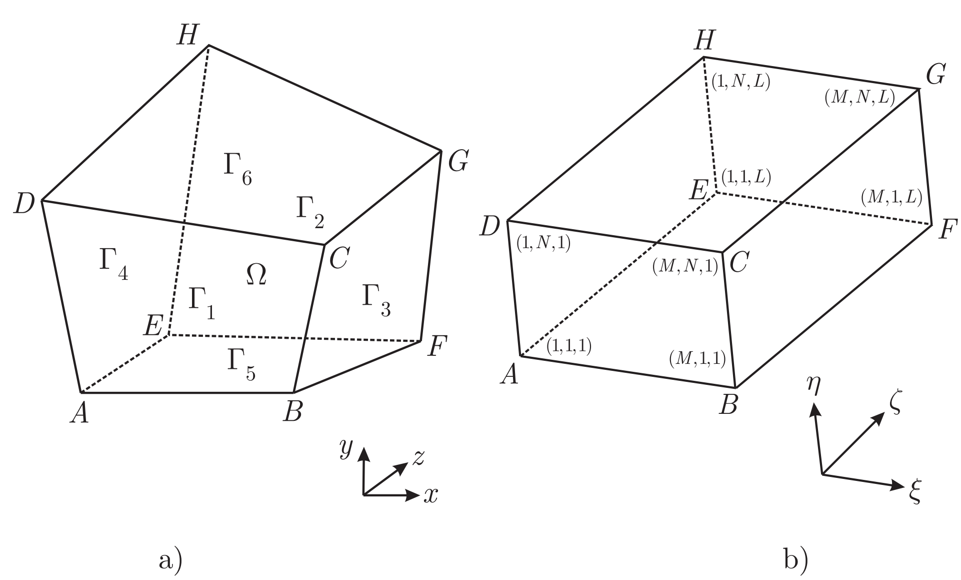

Fig. 1. Arbitrarily shaped heat-conducting body (physical domain) under specified boundary conditions (a). The 3D body is subjected to convective heat transfer on surfaces Γ_i, $i = 1$, 2, 3, 4, 6 and heat flux $\dot{q}$ on surface Γ_5. The thermal conductivity of the body is k_T. Computational domain corresponding to the physical domain (b).