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Optimal design of thermoelectric devices with dimensional analysis

HoSung Lee *

Mechanical and Aeronautical Engineering, Western Michigan University, 1903 W. Michigan Ave., Kalamazoo, MI 49008-5343, USA

HIGHLIGHTS

- ▶ This paper discusses optimal design of thermoelectric devices with dimensional analysis.
- ▶ A real design example for exhaust gas energy conversion was shown using the dimensional analysis.
- ▶ A real design example for automobile air conditioner was demonstrated using the dimensional analysis.

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ABSTRACT

The optimum design of thermoelectric devices (thermoelectric generator and cooler) in connection with heat sinks was developed using dimensional analysis. New dimensionless groups were properly defined to represent important parameters of the thermoelectric devices. Particularly, use of the convection conductance of a fluid in the denominators of the dimensionless parameters was critically important, which leads to a new optimum design. This allows us to determine either the optimal number of thermocouples or the optimal thermal conductance (the geometric ratio of footprint of leg to leg length). It is stated from the present dimensional analysis that, if two fluid temperatures on the heat sinks are given, an optimum design always exists and can be found with the feasible mechanical constraints. The optimum design includes the optimum parameters such as efficiency, power, current, geometry or number of thermocouples, and thermal resistances of heat sinks.

1. Introduction

Thermoelectric devices (thermoelectric generator and cooler) have found comprehensive applications in solar energy conversion [1], exhaust energy conversion [2,3], low grade waste heat recovery [4–6], power plants [7], electronic cooling [8], vehicle air conditioners, and refrigerators [7]. The most common refrigerant used in home and automobile air conditioners is R-134a, which does not have the ozone-depleting properties of Freon, but is nevertheless a terrible greenhouse gas and will be banned in the near future [9]. The pertinent candidate for the replacement would be thermoelectric coolers. Many analyses, optimizations, even manufacturers' performance curves on thermoelectric devices have been based on the constant high and cold junction temperatures of the devices. Practically, the thermoelectric devices must work with heat sinks (or heat exchangers). It is then very difficult to have the constant junction temperatures unless the thermal resistances of the heat sinks are zero, which is, of course, impossible.

A significant amount of research related to the optimization of thermoelectric devices in conjunction with heat sinks has been conducted as found in the literature [10–30]. It is well noted from the literature that there is the existence of optimal conditions in power output or efficiency with respect to the external load resistance for a thermoelectric generator (TEG) or the electrical current for a thermoelectric cooler (TEC). Many researchers attempted to combine the theoretical thermoelectric equations and the heat balance equations of heat sinks, and then to optimize design parameters such as the geometry of heat sinks [10], allocation of the heat transfer areas of heat sinks [12,13,18,19], thermoelement length [14], the number of thermocouples [15], the geometric ratio of the cross-sectional area of thermoelement to the length [16], and slenderness ratio (the geometric factor ratio of n-type to that of p-type elements) [17]. It can be seen from the above literature that the geometric optimization of thermoelectric devices is important in design and also formidable due to so many design parameters. The thermal conductance of thermoelements that is the most important geometric parameter has been often addressed in analysis, which is the product of three parameters: the number of thermocouples, the geometric ratio, and the thermal conductivity. In order to reduce the optimum design parameters, obviously dimensionless analyses were performed in the literature [21–26]. Yamanashi [21] developed optimum design introducing dimensionless parameters for a thermoelectric cooler with two heat

* Tel.: +1 269 276 3429 (O); fax: +1 269 276 3421.
E-mail address: hosung.lee@wmich.edu

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Nomenclature

A	cross-sectional area of thermoelement (cm^2)
A_1	total fin surface area at fluid 1 (cm^2)
A_2	total fin surface area at fluid 2 (cm^2)
A_b	base area of heat sink (cm^2)
COP	the coefficient of performance
h_1	heat transfer coefficient of fluid 1 (W/m^2 K)
h_2	heat transfer coefficient of fluid 2 (W/m^2 K)
I	electric current (A)
L	length of thermoelement (mm)
k	thermal conductivity (W/m K), $k = k_p + k_n$
n	the number of thermocouples
N_k	dimensionless thermal conductance $N_k = n(Ak/L)/\eta_2 h_2 A_2$
N_h	dimensionless convection, $N_h = \eta_1 h_1 A_1/\eta_2 h_2 A_2$
N_I	dimensionless current, $N_I = \alpha I/(Ak/L)$
N_V	dimensionless voltage, $N_V = V_n/(n\alpha T_{\infty 2})$
Q_1	the rate of heat transfer entering into TEG (W)
Q_2	the rate of heat transfer leaving TEG (W)
P_d	power density (W/cm^2)
R	electrical resistance of a thermocouple (Ω)
R_L	load resistance of a thermocouple (Ω)
R_r	dimensionless resistance, $R_r = R_L/R$
T_1	junction temperature at fluid 1 (°C)
T_2	junction temperature at fluid 2 (°C)
$T_{\infty 1}$	temperature of fluid 1 (°C)
$T_{\infty 2}$	temperature of fluid 2 (°C)
$T_{\infty 1,\max}$	maximum temperature of fluid 1 (°C)
$T_{\infty 1,\min}$	minimum temperature of fluid 1 (°C)
V_n	voltage of a module (V)
W_n	power output (W)
W_n	power input (W)
Z	the figure of merit

Greek symbols

α	Seebeck coefficient (V/K), $\alpha = \alpha_p - \alpha_n$
ρ	electrical resistivity (Ω cm), $\rho = \rho_p + \rho_n$
η_1	fin efficiency of heat sink 1
η_2	fin efficiency of heat sink 2
η_{th}	thermal efficiency of TEG

Subscripts

p	p-type element
n	n-type element
opt	optimal quantity
1/2*opt*	half optimal quantity

Superscript

$*$	dimensionless

sinks, wherein the thermal conductance appears twice in the nominators and fourth in the denominators of the dimensionless parameters. Although his work led to a new approach in dimensionless optimum design, the analysis encountered difficulties in optimizing the cooling power with respect to the thermal conductance because the conductance is intricately related to the others. Later, researchers [22,24,25] reported optimum design using the similar dimensionless parameters used by Yamanashi [21], presenting valuable optimum design features as Xuan [22] optimized cooling power for a TEC as a function of thermoelement length, Pan et al. [24] showed the optimum thermal conductance for a TEC with a given cooling power, and Casano and Piva [25] presented the optimum external load resistance ratio with heat sinks for a TEG which is greater than unity. There are also some experimental works [27–30] comparing with the theoretical thermoelectric equations. Gou et al. [27] conducted experiments for low-temperature waste heat recovery and demonstrated that the experimental results were in fair agreement with the solution formulas originally derived by Chen et al. [15] from the general theoretical thermoelectric equations. Chang et al. [28] and Huang et al. [29] conducted experiments for a TEC from a heat source with air-cooling and water cooling heat sinks, respectively. Casano and Piva [30] reported experimental work on a set of nine thermoelectric generator modules with a heat source on one side and a heat sink on the other side. After deliberately determined the heat leakage which turned out to be about 30% of the supplied heat source, they demonstrated that the theoretical performance curves of the power output and efficiency as a function of the external load resistance and temperature difference were in good agreement with the measurements. It is realized from the above experimental works that the theoretical thermoelectric equations with the heat balance equations of heat sinks can reasonably predict the real performance. However, proper optimum design still remains questionable.

In spite of many efforts for optimum design, its applications seem greatly challenging to system designers [1–3]. For example, Hsu et al. [3] in 2011 tested an exhaust heat recovery system both experimentally using an automobile and mathematically using computer simulations. They found a reasonable agreement between the measurements and the simulations. However, they obtained the power output of 12.41 W over 24 thermoelectric generator modules with the exhaust gas temperature of 573 K and the air temperature of 300 K. When the power output was divided by the footprint of 24 thermoelectric generator modules, it gives the power density of 0.032 W/cm^2, which seems unusually small. Karri et al. [2] in 2012 conducted a similar experiment with an SUV automobile. This time they designed the exhaust heat recovery system with an optimum coolant flow rate. They obtained the power output of 550 W over 16 thermoelectric generator modules with the exhaust gas temperature of 686 K and the coolant temperature of 361 K, which provided the power density of 0.61 W/cm^2. This shows a significant improvement, indicating the importance of optimum design. A New Energy Development Organization (NEDO) Program (Japan) [7] in 2003 also reported a similar experiment with a passenger car, obtaining the power output of 240 W over 16 segmented-type modules with the exhaust gas temperature of 773 K and the coolant temperature of 298 K, which provided the power density of ~1 W/cm^2. Notably, the power densities obtained are no way to evaluate how good it is until the better comes because proper optimum design seems not available.

From the review of the above theoretical and experimental studies including optimum design in the literature, it is summarized that the proper optimum design should be determined basically not only by the power output for TEG (or cooling power for TEC) but also by the efficiency (the coefficient of performance) simultaneously with respect to both the external load resistance (or the electrical current) and the geometry of thermoelement which refers to the number of thermocouples and the geometric ratio. The former (external load resistance) is well attained in the literature but the latter (geometry) is vague. Therefore, the optimum design seems incomplete. This is the rationale why the present paper is to improve the optimum design introducing new dimensionless parameters.