Applied Energy 104 (2013) 250–257

Contents lists available at SciVerse ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

# Optimal operation scheduling of a pumping station with multiple pumps

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## HIGHLIGHTS

- We formulate the optimal operation scheduling of a pumping station with multiple pumps as a dynamic programming problem.
- The extended reduced dynamic programming algorithm (RDPA) is proposed to solve the optimization problem.
- The extended RDPA reduces the admissible domain of the possible state values at each stage and that of the possible state transfer routes.
- The optimal scheduling strategy reduces the operational cost.
- The extended RDPA performs much more time efficient than the conventional DP algorithms.

## ARTICLE INFO

*Article history:*
Received 8 July 2012
Received in revised form 10 October 2012
Accepted 12 October 2012
Available online 17 December 2012

*Keywords:*
Optimal scheduling
Dynamic programming
Pumping station

## ABSTRACT

The optimal operation scheduling of a pumping station with multiple pumps is formulated as a dynamic programming problem. Based on the characteristics of the problem, an extended reduced dynamic programming algorithm (RDPA) is proposed to solve the problem. Both the energy cost and the maintenance cost are considered in the performance function of the optimization problem. The extended RDPA can significantly reduce the computational time when it is compared to conventional DP algorithms. Simulation shows the feasibility of the reduction of the operation cost.

## 1. Introduction

The operation of a pumping station is very important in achieving the tasks of the station. The main task is to maintain a suitable water volume in the reservoir and supply the demands. Another important task is to reduce the operation cost. With the operation scheduling optimized, remarkable reduction of the operation cost could be achieved while no change is needed with the physical elements, such as pumps and civil infrastructures [1].

The operation scheduling problem of a pumping station can be formulated as a cost optimization problem, of which the objective is to minimized the operation cost while the state constraints are satisfied. For a reservoir, the water volume/water level should be kept within a range to satisfy the security and operation requirements.

There are mainly two classes of operation costs for a pumping station. One is the energy cost, and the other is the maintenance cost. The maintenance cost, related to the wear of the rotating equipments, is difficult to be quantified. However it is true that the maintenance cost increases when the number of pump switches increases. A simple assumption is that the maintenance cost is proportional to the number of pump switches. Here a pump switch refers to "turning on a pump that was not operating in the previous period" [1].

Energy cost is the main part of the operation cost. When physical elements are not changed, the energy cost is related to the energy consumption and the energy pricing structure. The energy consumption is proportional to the pump power and the operational time length. With the time of use (ToU) electricity pricing structure implemented, the operation scheduling has a heavy influence on the energy cost if there is room for the scheduling of the pump operation. Such kind of load shifting has been studied for various systems, such as in [2] for steel plants and in [3] for a deep-mine water reticulation system.

The problem on the optimal operation scheduling of a pumping station has been studied in many papers in recent years. The problem for a pumping station with fixed-speed motored pumps is intrinsically an integer programming problem (linear or nonlinear), depending on the mathematical models of the hydraulic structures, networks, etc. For such a kind of integer programming problems, various techniques have been employed in load shifting

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http://dx.doi.org/10.1016/j.apenergy.2012.10.028

for different processes. The linear programming is employed in [4] for a wind/hydro hybrid water supply system while the dynamic programming (DP) in [5–7] for a water supply system and a pump station. The stochastic DP is considered in [8] for a water supply system with the water demand modeled as a Markov process. The binary integer programming (BIP) is used in [9] for the operation scheduling of a colliery. The above methods can theoretically solve the optimal operation scheduling problems, but they are limited in practice when the underlying model is large or complex because of *curse of dimensionality* of the DP or *interminable branch and bound* of the integer programming.

To search the global optimal solution to a programming problem, some modern optimization methods, such as genetic algorithm [10–13], simulated annealing [14,15], particle swarm optimization [16], ant colony optimization [17] and fuzzy optimization [18], are adopted. Those approaches improve the possibility of obtaining the global optimization solution while the computational time is sometimes very long and the algorithms are sometimes too complex, which again limits their application.

In [19], a reduced dynamic programming algorithm is developed to address the optimal operation scheduling problem with the capability of fast computation. The scheduling problem is reformulated as a control sequence optimal scheduling problem. This algorithm is a cost-efficient scheduling approach for the pump operation.

The optimal operation scheduling in [19], is studied for a pumping station with only one pump. The admitted domain of the control variable is $\{0,1\}$. When more pumps' operation is required to be optimized, the method in [19] could not be directly employed. The optimal operation scheduling of a pumping station with multiple pumps are considered in this paper. The approach of RDPA is re-investigated and extended to implement in a pumping station with multiple pumps. The problem studied here for a pumping station with multiple pumps is different from the one in [19] in the following aspects.

(1) The domain of the control variable is larger (the domain is $\{0,1,\ldots,N_p\}$, where $N_p$ is the number of pumps).
(2) The number of possible values for the water volume at the $s$th stage is larger. For one pump, the number is $s+1$ while for multiple pumps, it is $s \times N_p + 1$.
(3) The number of possible routes from the $(s-1)$th stage to the $s$th stage is larger, too. For one pump, the number is at most two while for multiple pumps, it can be as large as $N_p + 1$.
(4) The maintenance cost is considered in the operation cost.

Compared with the conventional DP algorithm, the nature of fast computation of the extended RDPA is owing to two aspects. One is that the number of possible values at stage $s$ is reduced from $s \times N_p + 1$ with RDPA to $k_s^u - k_s^l + 1$, which is much less than $s \times N_p + 1$ when $s$ is large. The other is that the number of state transfer routes and the comparison of the cost function at stage $(s+1)$ are significantly reduced to less than $(sN_p+1)(N_p+1)$ from $(sN_p+1)((s-1)N_p+1)$ with a conventional DP algorithm.

Simulation shows the feasibility of extended RDPA in the reductions of the energy cost and the number of pump switches. Both the penalty on a pump switch and the time step of the scheduling in extended RDPA have influences on the number of pump switches.

The main contributions of this paper are: (1) The optimal operation scheduling of a pumping station with multiple pumps is formulated as a dynamic programming problem; (2) The maintenance cost can be explicitly considered in the cost function; (3) The RDPA in [19] is extended to solve the problem of a pumping station with multiple pumps.

The structure of this paper is: in Section 2, the optimal operation scheduling problem of a pumping station with multiple pumps is formulated under the ToU electricity tariff structure, followed by the extension of the RDPA in Section 3. Simulation of the extended RDPA for a pumping station with three pumps is given in Section 4. Some conclusions are give in Section 5.

## 2. Problem formulation

A typical reservoir with a pumping station is shown in Fig. 1. There are several pumps in the station. Generally, the pumps are identical, including the outflow capacity and the corresponding power. The pump's outflow capacity and the power are denoted as $b$ and $P_m$, respectively.

The water volume in the reservoir is $\mathcal{V}(t)$, which satisfies the following equation,

$$\dot{\mathcal{V}}(t) = a - b\sum_{i=1}^{N_p} u_i(t), \tag{1}$$

where $a$ is the inflow rate and $u_i(t)$ is the $i$th pump state in the station, which can be either one for the pump being on or zero for the pump being off. Then the sum $\sum_{i=1}^{N_p} u_i(t)$ is an integer denoting the number of pumps being 'on' in the station at time $t$.

The electricity price structure is as follows,

$$P_e(t) = C(p), t \in [T_e^p, T_e^{p+1}], \quad p = 1, \ldots, P$$

where $P$ is the number of the time intervals within the time period $[t_0, t_f]$ and in each of the intervals the electricity price is constant.

The ToU electricity rate can differ by time of day, day of week and season. A typical ToU electricity pricing structure is shown in Fig. 2, which shows the rate by time of a week with off-peak, mid-peak and on-peak intervals given in Table 1.

Generally, an optimal operation scheduling problem of the pumping station is: to find, a control sequence $\{U(k)\}$ $(U(k) = [u_1(k), u_2(k), \ldots, u_{N_p}(k)]^T)$ and the corresponding switching time sequence $\{t(k)\}$ such that the energy cost within the time period $[t_0, t_f]$ is minimized with the water volume constraints $\mathcal{V}(t) \in [\mathcal{V}^l, \mathcal{V}^u]$ satisfied. Here $u_i(k)$ is either zero or one within the time interval $[t(k), t(k+1)]$.

Such a problem is reformulated in [20] as a control sequence optimization problem when the time sequence $\{t(k)\}$ is given. Especially, when $t(k) = t_0 + k \times T_{\text{sampling}}$, $k = 0, \ldots, K$ where $T_{\text{sampling}}$ is the sampling time period and constant, the problem is intrinsically a BIP problem.

Within the framework of the dynamic programming problem studied here, a given time period $[t_0, t_f]$, for example, the period of the ToU pricing, could be scheduled into $S$ subintervals according to the ToU electricity pricing structure and the sampling time period $T_{\text{sampling}}$. Within each time subinterval, the time length of the subinterval is $T_{\text{sampling}}$ and the electricity price is constant.

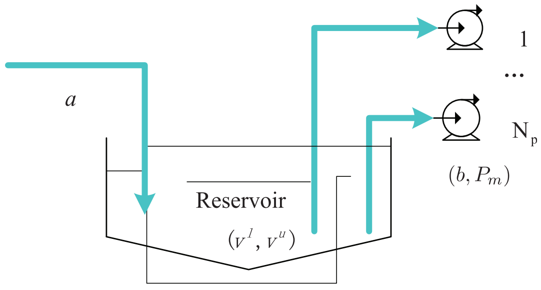

**Fig. 1.** A typical reservoir with a pumping station.