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Acute anisotropic scattering in a medium under collimated irradiation

A. Collin [a,b,*], J.L. Consalvi [b], P. Boulet [c]

[a] LEMTA, Nancy-Université, CNRS, ENSEM - 2 Avenue de la Forêt de Haye, BP 160, 54504 Vandoeuvre lès Nancy Cedex, France
[b] Polytech'Marseille, UMR CNRS 6595, Technopôle de Château-Gombert, 5 Rue Enrico Fermi, 13453 Marseille Cedex 13, France
[c] LEMTA, Nancy-Université, CNRS, Faculté des Sciences et Techniques, BP 70239, 54506 Vandœuvre lès Nancy Cedex, France

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ABSTRACT

This article deals with the assessment of the Finite Volume Method (FVM) to solve the radiative transfer equation (RTE) in 3D acute forward-scattering media with collimated irradiation. Phase functions of the Henyey-Greenstein (HG) type with asymmetry factors ranging from 0.2 to 0.93, and a Mie phase function with an asymmetry factor of 0.93, are considered. The test case involves a purely scattering medium in a cubic enclosure, with a collimated irradiation with zero incidence impinging on one face. FVM results are found to be consistent with reference solutions obtained by the Monte Carlo Method (MCM) whatever the magnitude of the forward peak and the optical thickness considered. The FVM appears to be a good candidate to treat such problems since it requires no modification of its classical formulation, resulting in a straightforward implementation.

1. Introduction

The capacity of the Discrete Ordinates Method (DOM) [1] and the Finite Volume Method (FVM) [2,3] to predict radiative transfer in multidimensional anisotropic scattering media under both diffuse or collimated incidence was demonstrated using smooth phase functions such as the well-known B1, B2, F1 and F2 functions. These functions, derived from a modified Mie formulation, are able to describe both forward and backward scattering but they are limited to size parameters of less than 5 (see [1] for example for a description of these functions). The forward-scattering peaks are then less acute than those encountered in certain applications such as water mists, where size parameters can reach really high values of up to several hundreds. In a recent study [4] the DOM and the FVM were tested in the case of acute forward-scattering media under diffuse incidence. Four phase functions were considered: three of the Henyey–Greenstein type with asymmetry factors of 0.2, 0.8 and 0.93 and a Mie phase function with a strong forward peak (corresponding to an asymmetry factor of 0.93 in a realistic case taken from a mist application with a size parameter of 245). The results revealed that the DOM combined with the classical procedures of the renormalization of the phase function proposed by Kim and Lee [1,5] and Wiscombe [6] becomes inaccurate when the phase function exhibits a strong forward peak. For such scattering media the renormalization techniques induce a deformation of the discretized phase function, leading to an overestimation of the forward scattering. This inaccuracy increases with the asymmetry factor and the optical thickness. In some cases with a really acute forward peak the attenuation by scattering was found not to be taken into account at all. Possible improvement of the results obtained with the DOM can be sought using a Delta-Eddington approximation. This approximation consists in separating forward scattering from effective scattering. It does not require a renormalization procedure when effective scattering is approximated as isotropic or linear anisotropic scattering. Although this solution is often applied in acute anisotropic scattering the maximum relative error was found to reach 20% when compared with the Monte Carlo Method (MCM) solution [4]. The FVM [2,7] was found to be accurate whatever the phase function and the optical thickness considered with a maximum relative error of less than 3%. With this method, the discretized values of the phase function represent the averaged part of the scattered energy from one control angle to another, which ensures the conservation of the scattered energy.

This avoids a renormalization procedure and the actual phase function shape is preserved in an average sense.

The FVM was thus found to be more appropriate than the DOM to predict radiative heat transfer in medium with acute forward scattering.

* Corresponding author. LEMTA, Nancy-Université, CNRS, ENSEM - 2 Avenue de la Forêt de Haye, BP 160, 54504 Vandoeuvre lès Nancy Cedex, France. Tel.: +33 3 83 59 55 55.

E-mail address: anthony.collin@ensem.inpl-nancy.fr (A. Collin).

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Nomenclature

a	coefficient in the discretization equation, [m^2sr]
b	source term in the discretization equation, [W]
D_c^l	direction cosine integrated over $\Delta\Omega^l$, [sr]
$\mathbf{e}$	unit direction vector, [-]
g	asymmetry factor, [-]
I	radiative intensity, [$Wm^{-2}sr^{-1}$]
L	total number of angular directions, [-]
m	complex optical index, [-]
$\mathbf{n}$	unit normal vector, [-]
q	radiative flux density, [Wm^{-2}]
s	path length, [m]
S_m^l	modified source function, [$Wm^{-3}sr^{-1}$]
T	temperature, [K]
x,y,z	coordinate directions, [m]

Greek symbols

β_m^l	modified extinction coefficient, [m^{-1}]
ΔA	area of control-volume faces, [m^2]
$\Delta\upsilon$	volume of a control volume, [m^3]
$\Delta\Omega^l$	control angle, [sr]
θ	polar angle measured from the z-axis, [rad]
σ	scattering coefficient, [m^{-1}]
φ	azimuthal angle measured from the x-axis, [rad]
Φ	scattering phase function, [-]
Ω	directional vector of radiative intensity, [-]

Superscripts

l,l'	angular directions

Subscript

B	blackbody
e,w,n,s,f,b	east, west, north, south, front and back control volume faces
E,W,N,S,F,B	east, west, north, south, front and back neighbors of P
P	central grid point under consideration
w	wall

List of abbreviations

FVM	Finite Volume Method
HG	Henyey and Greenstein
MCM	Monte Carlo Method
RTE	Radiative Transfer Equation

The present study complements reference [4] by assessing the efficiency of the FVM in predicting radiative heat transfer in really acute forward anisotropic scattering medium under collimated incidence. Such a situation is encountered in many topics such as the attenuation of fire radiation by water mist or the use of sprays as radiative shields, the combustion in fuel sprays, the interaction between solar radiation and atmosphere, the characterization of porous media under collimated incidence,… In these problems directional effects of scattering can be more difficult to address. To the authors best knowledge an assessment of the FVM in such particular configurations has not been carried out in the past. Comparisons will be made between results obtained with the FVM and reference data yielded by the MCM in a case of a purely scattering medium in an enclosure. The numerical methods are described in Section 2. Section 3 discusses the results concerning the radiative flux predictions for a purely scattering case. Finally, Section 4 presents the conclusions deduced from the current study.

2. Numerical methods

The present problem concerns a cubic domain containing a purely scattering medium with cold black boundaries, except on one face assumed to be transparent, through which an external collimated radiation penetrates the medium. The only case of a perpendicular collimated incidence Ω_c will be addressed numerically for illustration purposes (see Fig. 1).

2.1. Radiative transfer equation (RTE)

Considering a general formulation of the RTE for a purely scattering medium, the radiative transfer problem is modeled as follows:

$$\frac{dI}{ds}(s,\Omega) = -\sigma I(s,\Omega) + \int_{4\pi} \sigma I(s,\Omega')p(\Omega' \to \Omega)d\Omega' \tag{1}$$

The boundary condition for Eq. (1) is for any location s_w on the transparent surface:

$$I(s_w,\Omega) = q_w\delta(\Omega - \Omega_c) \tag{2}$$

where δ is the classical Dirac-delta function, [8]. Emission from the surrounding medium and diffuse reflection at the interface are assumed to be negligible.

2.2. Finite Volume Method (FVM)

The flexibility of the angular discretization used with the FVM [9] allows a straightforward formulation of the collimated incidence problem. The angular mesh can be refined around the incidence direction in order to capture the collimated beam.

The discretization of the RTE in its integral form given by Eq. (1) has been described by many authors and is not repeated here. The final discretized form of the RTE for a general control volume and control angle can be written as

$$a_P^l I_P^l = a_w^l I_W^l + a_e^l I_E^l + a_s^l I_S^l + a_n^l I_N^l + a_b^l I_B^l + a_f^l I_F^l + b^l \tag{3}$$

When using the STEP scheme, the different coefficients in the previous equation are given by

$$a_i^l = \max\left(-\Delta A_i D_{ci}^l, 0\right), i = w, e, s, n, b, f \tag{4}$$

$$a_P^l = \sum_{i=1}^{6} \max\left(\Delta A_i D_{ci}^l, 0\right) + \left(\beta_m^l\right)_P \Delta\upsilon\Delta\Omega^l \tag{5}$$

$$b^l = \left(S_m^l\right)_P \Delta\upsilon\Delta\Omega^l \tag{6}$$

$$D_{ci}^l = \int_{\Delta\Omega^l} \left(\Omega^l \cdot \mathbf{n}_i\right) d\Omega \tag{7}$$