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Local thermal non-equilibrium flow with viscous dissipation in a plane horizontal porous layer

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ABSTRACT

Darcy's flow in a horizontal porous layer with impermeable boundaries is studied. The viscous dissipation effect is taken into account and the local thermal non-equilibrium (LTNE) model for the energy balance is adopted. The upper boundary is assumed to be perfectly isothermal and the lower boundary is taken to be thermally insulated. The basic solution is expressed analytically. The case of a perfectly conducting solid phase is considered. The onset of convective roll instability is investigated by a linear analysis, with different values of the inter-phase heat transfer parameter. The eigenvalue problem is solved numerically by a Runge–Kutta method.

1. Introduction

Viscous dissipation can play an important role in the stability analysis of basic flow solutions in porous media. In this kind of problems, a sufficiently intense temperature gradient is needed for the onset of convective instabilities. In the absence of a thermal forcing induced by the temperature boundary conditions, the viscous dissipation effect may be the only possible cause of instability. For instance, in a horizontal porous layer with an upper isothermal boundary and a lower adiabatic boundary, a possibly unstable stratification may be induced by the frictional heating associated with a basic horizontal throughflow. In the classical Darcy–Bénard problem [1–4], the basic temperature gradient is forced by the boundary conditions and the viscous dissipation provides a nonlinear contribution or, more precisely, a second order term in the perturbations. The latter term, in a linear stability analysis, is neglected. On the other hand, if we consider a Prats-like problem [5], a basic horizontal throughflow is imposed and the viscous dissipation provides also a linear term in the perturbations and, thus, it may influence the onset conditions of the instability. A wide work has been done in the last decades for investigating the role played by the viscous dissipation in the convection processes occurring within a porous medium [6]. Nakayama and Pop [7] showed that the effect of viscous dissipation results into a reduction of the heat transfer rate between a non-isothermal body and a surrounding fluid saturated porous medium. Nield [8] resolved a paradox in the modelling of the viscous dissipation term for convective flows in porous media described through the Darcy–Forchheimer momentum transfer law. The effect of viscous dissipation in the forced convection heat transfer was studied with reference either to a parallel plate channel [9] or to a circular duct [10] filled with a porous medium.

The lack of local thermal equilibrium in porous flows may occur not only in unsteady conditions, but also in a stationary regime. Conditions of local thermal non-equilibrium (LTNE) [1,11–15] between the fluid phase and the solid phase may take place, for instance, when there are strong differences between the thermal conductivities of the two phases. In these cases, the averages over a representative elementary volume of the fluid temperature and of the solid temperature may yield different values for the two phases. Beyond this example, cases of LTNE with small differences between the thermal conductivities of the two phases may be conceivable. Thus, the LTNE approach to the description of convective flows is based on two distinct temperature fields for the fluid phase and for the solid phase, defined as the solutions of two local energy balances. These energy balance equations are coupled through exchange terms modelled by means of a microscopic Newton's cooling law between the phases. This model has been applied in an LTNE analysis of the classical Darcy–Bénard problem [16], in the study of the evolution of a thermal front in a semi-infinite fluid

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Nomenclature

a	dimensionless wave number, Eq. (37)
c	heat capacity per unit mass
$\boldsymbol{e}_y$	unit vector in the y-direction
$F(y,H)$	dimensionless function, Eq. (41)
$\boldsymbol{g}$,g	gravitational acceleration; modulus of $\boldsymbol{g}$
Ge	Gebhart number, Eq. (8)
h	inter-phase heat transfer coefficient
H	dimensionless inter-phase heat transfer parameter, Eq. (8)
k	thermal conductivity
L	layer thickness
Pe	Péclet number, Eq. (18)
$\tilde{Q}$	dimensionless average heat exchanged, Eq. (25)
R	dimensionless parameter, $GePe^2$
T	dimensionless temperature
T_0	constant temperature
$\tilde{T}_{fB}$	average temperature of the fluid phase, Eq. (24)
$\boldsymbol{u}$	dimensionless velocity, (u,v,w)
$\boldsymbol{U}$	dimensionless velocity disturbance, (U,V,W)
$\boldsymbol{x}$	dimensionless position vector, (x,y,z)

Greek Symbols

α	thermal diffusivity
β	thermal expansion coefficient
ε	dimensionless perturbation parameter
η	dimensionless parameter, Eq. (48)
θ	dimensionless fluid phase temperature disturbance
Θ, Φ, Ψ	dimensionless disturbance amplitudes, Eq. (37)
Λ	dimensionless parameter, Eq. (8)
μ	dynamic viscosity
ξ	dimensionless similarity variable, Eq. (49)
ρ	density
φ	porosity
ϕ	dimensionless solid phase temperature disturbance
χ	inclination angle, Eq. (15)
ψ	dimensionless streamfunction disturbance
ω	dimensionless angular frequency, Eq. (37)
Ω	dimensionless parameter, Eq. (8)

Superscript, subscripts

$-$	dimensional quantity
^	rescaled dimensionless quantities, Eq. (51)
B	basic flow
cr	critical value
f	fluid phase
s	solid phase

saturated porous medium [17], the unsteady injection of a fluid with a uniform temperature profile in a porous medium [18], the steady free convection in a two-dimensional square cavity with side heating [19]. An extension of the Darcy–Bénard problem with LTNE and a non-isotropic permeability model of the porous medium has been carried out by Malashetty et al. [20]. Recently, an analysis of the Prats problem [5] under conditions of LTNE has been performed by Postelnicu [21].

In the present paper, a linear stability analysis of a basic Darcy's flow in a horizontal porous layer with impermeable boundaries is studied. The viscous dissipation term in the energy balance for the fluid phase is taken into account [22,23]. Two different temperature fields for the porous solid and for the saturating fluid are assumed in order to model the LTNE. Two local energy balances, one for each phase, are introduced. The upper boundary is taken to be perfectly isothermal and the lower boundary is assumed to be thermally insulated. In this configuration, the viscous dissipation contribution provides a source of possible instability. The basic velocity field is assumed to be stationary and uniform. The basic solution is expressed analytically and perturbed by means of plane waves in order to investigate the onset of convective rolls. The special case of a porous solid with a very high thermal conductivity is examined. This assumption is a sensible one when the flow through a metallic foam is considered. The eigenvalue problem thus obtained is solved by means of a Runge–Kutta method. The onset of the instability is described through the governing dimensionless parameters H and R, where H is the inter-phase heat transfer parameter and R is the stability parameter defined as $R = GePe^2$. Here, Ge is the Gebhart number and Pe is the Péclet number.

2. Mathematical model

We study a horizontal porous layer with impermeable boundary planes $\bar{y} = 0$ and $\bar{y} = L$. The $\bar{y}$-axis is oriented upward, so that $\boldsymbol{g} = -g\boldsymbol{e}_y$. The lower boundary, $\bar{y} = 0$, is thermally insulated, while the upper boundary $\bar{y} = L$ is kept at a uniform temperature T_0. A sketch of the porous layer is reported in Fig. 1. Let us assume that:

- Darcy's law holds;
- the Oberbeck–Boussinesq approximation can be applied;
- the viscous dissipation cannot be neglected;
- a condition of LTNE holds.

Then, the governing balance equations can be written as

$$\overline{\boldsymbol{\nabla}}\cdot\overline{\mathbf{u}} = 0, \tag{1}$$

$$\frac{\mu}{K}\overline{\boldsymbol{\nabla}}\times\overline{\mathbf{u}} = \rho_f g\beta\overline{\boldsymbol{\nabla}}\times\left(\overline{T}_f - T_0\right)\mathbf{e}_y, \tag{2}$$

$$(1-\varphi)(\rho c)_s\frac{\partial\overline{T}_s}{\partial\bar{t}} = (1-\varphi)k_s\overline{\nabla}^2\overline{T}_s + h\left(\overline{T}_f - \overline{T}_s\right), \tag{3}$$

$$\varphi(\rho c)_f\frac{\partial\overline{T}_f}{\partial\bar{t}} + (\rho c)_f\overline{\mathbf{u}}\cdot\overline{\boldsymbol{\nabla}}\overline{T}_f = \varphi k_f\overline{\nabla}^2\overline{T}_f + \frac{\mu}{K}\overline{\mathbf{u}}\cdot\overline{\mathbf{u}} + h\left(\overline{T}_s - \overline{T}_f\right), \tag{4}$$

where the curl operator has been applied to the local momentum balance equation in order to eliminate the pressure gradient term.

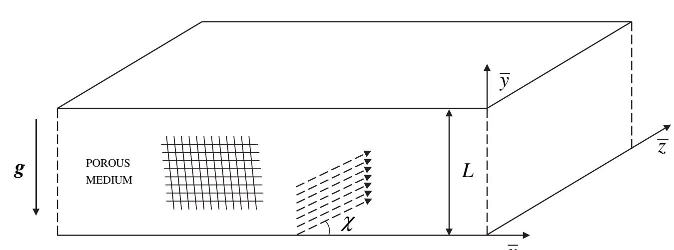

Fig. 1. Drawing of the porous layer and of the basic flow.