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A note on natural convection along a convectively heated vertical plate

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ABSTRACT

The steady laminar boundary layer flow along a vertical stationary plate with convective surface boundary condition is investigated in this paper. The heat transfer coefficient is considered either constant or variable along the plate and the problem is either non-similar or similar. The results are obtained with the direct numerical solution of the governing equations. The problem is governed by Prandtl number and a convective parameter and the influence of these parameters on the results are presented in tables and figures. There are differences in the results between the non-similar and similar case at low values of the convective parameter but as this parameter increases the differences decrease and the flow tends to the classical natural convection along a vertical isothermal plate.

1. Introduction

Aziz [3] was probably the first who treated the convective heat transfer on a horizontal plate in a constant free stream (Blasius flow) proving that a similarity solution is possible if the convective heat transfer of the fluid heating the plate on its lower surface is proportional to $x^{-1/2}$. If the convective heat transfer coefficient is constant then the problem does not admit a similarity solution.

The appearance of the above paper stimulated a large number of subsequent papers concerning different boundary layer problems with convective boundary conditions. See for example boundary layer flow with entropy generation [5], in power-law fluids [10], in micropolar fluids [12], with variable suction [9], in nanofluids [4], with variable fluid properties [22], in MHD flows [6], with heat and mass transfer [8] and with exact solution [11] to mention just a few of them.

All works mentioned above concern a convective heat transfer coefficient as a function x in order that the problems accept similarity solutions. However, the assumption of a heat transfer coefficient varying along the plate as a function of x is not realistic and very difficult to be obtained in practice. For that reason, it could be said, that the above works have only theoretical value. In contrast to this hypothesis Merkin and Pop [13] presented a non-similar solution of the Blasius flow with constant heat transfer coefficient. In the present work we treat the classical natural convection along a vertical stationary plate in a calm fluid either with constant heat transfer coefficient or with heat transfer coefficient variable along the plate as a function of $x^{-1/4}$. In the first case the problem is non-similar whereas in the second case we have a similarity solution. It is reminded here that the first approximate solutions to the problem of natural convection along a vertical isothermal plate was given by Schuh in 1948 [23] whereas Ostrach [15] numerically obtained the solution for the Pr range from 0.01 to 1000. The solutions given by Ostrach are included in many heat transfer and fluid mechanics textbooks, (see for example Refs. [25]; page 324, [24]; page 274).

2. Problem definition and solution procedure

Consider the flow along a vertical semi-infinite plate with u and v denoting respectively the velocity components in the x and y directions, where x is the coordinate along the plate and y is the coordinate perpendicular to x. For a steady, two-dimensional flow, the boundary layer equations are

$$\text{continuity equation}: \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\text{momentum equation}: u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty) \tag{2}$$

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$$\text{energy equation}: u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2} \quad (3)$$

The boundary conditions are:

$$\text{at } y = 0: u = 0, \ v = 0, \ -k\frac{\partial T_w(x)}{\partial y} = h(x)\left(T_f - T_w(x)\right) \quad (4)$$

$$\text{as } y \to \infty \ \ u = 0, \ T = T_\infty \quad (5)$$

where υ is the fluid kinematic viscosity, a is the fluid thermal diffusivity, k is the fluid thermal conductivity, T is the fluid temperature, T_w is the plate temperature (variable) and T_∞ is the ambient fluid temperature. It is assumed that the plate is heated by convection from a fluid with constant temperature T_f with a heat transfer coefficient h.

The Equations (1)–(3) represent a two-dimensional parabolic problem. Such a flow has a predominant velocity in the streamwise coordinate which in our case is the direction along the plate. In this type of flow convection always dominates the diffusion in the streamwise direction. Furthermore, no reverse flow is acceptable in the predominant direction. The solution of this problem in the present work is obtained using a finite difference algorithm as described by Patankar [21]. In order to obtain a complete form of both the temperature and velocity profile at the same cross section we used a nonuniform lateral grid. Δy takes small values near the surface (dense grid points near the surface) and increases along y. A total of 500 lateral grid cells were used. It is known that the boundary layer thickness changes along x. For that reason the calculation domain must always be at least equal to or wider than the boundary layer thickness. In each case we tried to have a calculation domain wider than the real boundary layer thickness. This has been done by trial and error. If the calculation domain was thin the velocity and temperature profiles were truncated. In this case we used another wider calculation domain in order to capture the entire velocity and temperature profiles. The parabolic (space marching) solution procedure is described analytically in the textbook of Patankar [21] which "remains to this day a model of simplicity and clarity and one of the most coherent explications of the finite volume technique ever written" [1]. The above solution procedure is implicit and unconditionally stable ([25]; page 276), has been used extensively in the literature and has been included in fluid mechanics and heat transfer textbooks (see Refs. [2]; p. 364, [25]; p. 271; and [14]; p. 124). The method has been used successfully in a series of papers by the present author [16–20].

3. Results and discussion

The problem is governed by two non-dimensional parameters, the Prandtl number and the convective parameter which are defined as

$$Pr = \frac{\upsilon}{a} \quad (6)$$

$$N_c = \frac{hx}{k}Gr_X^{-1/4} \quad (7)$$

where Gr_X is the classical Grashof number defined as

$$Gr_X = \frac{g\beta\left(T_f - T_\infty\right)x^3}{\upsilon^2} \quad (8)$$

Following the work of Aziz and Khan [4] we found that if the heat transfer coefficient h is proportional to $x^{-1/4}$, N_c becomes independent of x and a true similarity is realized. If h in Equation (7) is constant the convective parameter N_c is variable along the plate and the problem is non-similar.

Important parameters for this problem are the non-dimensional wall shear stress and the non-dimensional wall heat transfer defined, according to classical natural convection along a vertical isothermal plate ([25]; page 324, [24]; page 274) as

$$f''(0) = \frac{x^2}{(2^{1/2}\upsilon)}Gr_X^{-3/4}\left[\frac{\partial u}{\partial y}\right]_{y=0} \quad (9)$$

$$\vartheta'(0) = -\frac{x}{\left(T_f - T_\infty\right)}\left[\frac{Gr_X}{4}\right]^{-1/4}\left[\frac{\partial T}{\partial y}\right]_{y=0} \quad (10)$$

In addition the following relations are valid

$$\vartheta = \frac{T - T_\infty}{T_f - T_\infty} \quad (11)$$

$$f' = \frac{ux}{2\upsilon}Gr_X^{-1/2} \quad (12)$$

$$\eta = \frac{y}{x}\left[\frac{Gr_X}{4}\right]^{1/4} \quad (13)$$

In Fig. 1 the influence of the number of lateral grid points on the velocity profile for $Pr = 1$ and $N_c = 1$ is shown for the non-similar problem. According to Equation (7) the quantity N_c can be considered as a non-dimensional distance along the plate. It is seen that as the number of grid points increases the velocity profiles become better and for $m = 50$ the velocity profile is close to the final profile. The 500 lateral grid points, used in the present work, cover all the cases treated and thus the results of the present work are grid independent. The corresponding grid tests for the temperature profile are shown in Fig. 2 where it is seen that the 500 lateral grid points give also a very accurate temperature profile.

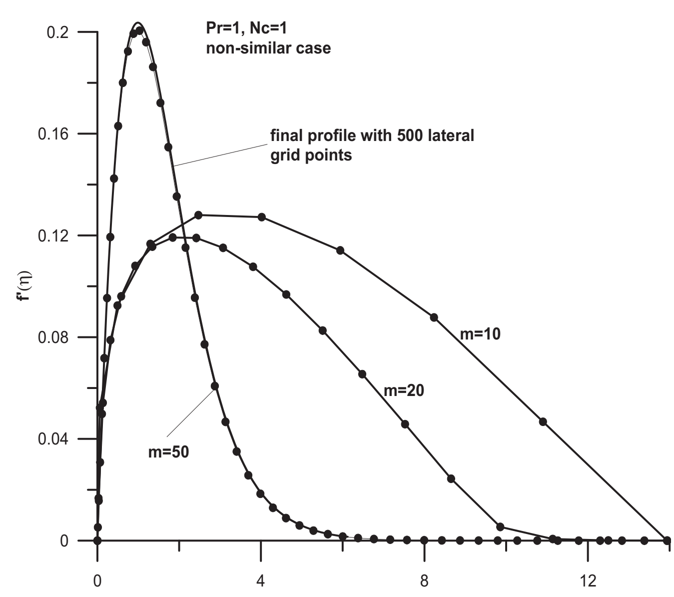

Fig. 1. Grid independent test for velocity profile for $Pr = 1$ at the non-dimensional distance $N_c = 1$ along the plate. m is the number of grid points across the boundary layer.