Applied Energy 101 (2013) 253–260

Contents lists available at SciVerse ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

A mathematical model to develop a Scheffler-type solar concentrator coupled with a Stirling engine

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HIGHLIGHTS

- ▶ Mathematical model to develop a Scheffler-type solar concentrator.
- ▶ Equation to estimate the intercept factor.
- ▶ Comparisons between the parabolic dish and a Scheffler-type solar concentrator.
- ▶ Proposed Scheffler-type solar concentrator with advantages over parabolic dish concentrator.

ARTICLE INFO

Article history:
Received 10 December 2011
Received in revised form 9 May 2012
Accepted 25 May 2012
Available online 22 June 2012

Keywords:
Solar energy
Stirling engine
Intercept factor
Parabolic dish
Scheffler concentrator
Mathematical model

ABSTRACT

This study develops and applies a new mathematical model for estimating the intercept factor of a Scheffler-type solar concentrator (STSC) based on the geometric and optical behaviour of the concentrator in Cartesian coordinates, and the incorporation of a thermal model of the receptor is performed using numerical examinations to determine the technical feasibility of attaching the STSC to a 3 kWe Stirling engine. A numerical validation of the mathematical model is determined based on the experimental results reported for the WGA500 concentrator and the CNRS-PROMES system receiver. The numerical results allow for the design of the STSC and a comparison with a parabolic dish that provides the same thermal demand. Our findings show that the highest concentration was obtained with an edge angle of 45°, which was observed in the parabolic dish as well, but the STSC receiver shows a 7% increase in the thermal efficiency compared with the efficiency of the parabolic dish receiver. Finally, the STSC is appropriate for regions where the solar height allows for a reduction of convective thermal loss.

1. Introduction

When generating electricity using alternative energy sources, it is appealing to incorporate technologies that concentrate solar thermal energy, such as heliostats, parabolic troughs and parabolic dishes connected to a Stirling engine [1]. The latest solar thermal technologies have high costs of installation, operation and maintenance and a decreased solar energy-to-electricity conversion efficiency [2]. Having a Stirling engine at the focal point of a parabolic dish decreases the effects of this efficiency problem by incorporating a solar concentrator with a fixed focal point. Scheffler developed a solar concentrator for solar cookers that are fixed inside the house [3]. This concentrator also has other applications, such as oil extraction [4], distillation [5] and sterilisation applications [6]. To incorporate this technology to generate electricity using Stirling engines, it is necessary to make some adjustments, such as incorporating an azimuth axis that tracks the solar height and form continuously and incorporates the reflector surface with the least amount of imperfections, which is motivated by a study of ray tracing [7].

Different mathematical models have been developed using various mathematical tools and software to optimise the design of assembly systems. The model proposed by Harris and Lenz [8] analysed the thermal behaviour of a parabolic dish concentrator that used the view factor to estimate the amount of radiation reaching the receiver cavity in cylindrical, conical, elliptical and spherical receivers. Additional results for various concentrator geometrical shapes have been provided in Badescu [9]. Shuai et al. [10] used the Monte Carlo method to determine the performance of the radiation concentrated by a parabolic dish receiver for various geometries. Kumar and Reddy [11] presented a software implementation of a CFD (computational fluid dynamics) study to determine the optimal size of the aperture opening area of a spherical cavity coupled to a parabolic dish, which depends on the item's minimum convection. Chin-Hsiang and Hang-Suin [12] presented a study to

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http://dx.doi.org/10.1016/j.apenergy.2012.05.040

Nomenclature

A	area (m^2)
a	normal distribution segment (–)
b	constant approximation to normal distribution (–)
C_{geo}	geometric concentration (–)
d	diameter (m)
f	focal length (m)
Gr	Grashof number (–)
h	convective heat transfer coefficient (W/m^2 K)
I	direct solar irradiance (W/m^2)
K	fluid Thermal Conductivity (W/(m K))
L	thickness length (m)
p	distance from concentrator surface to focal point (m)
Pr	Prandtl number (–)
n	number of segment reflectors (–)
Nu	Nusselt number (–)
Q	energy flux density (W/m^2)
r	radius (m)
Ra	Rayleigh number (–)
Re	Reynolds number (–)
Sp	spacing (m)
S	separation (m)
T	temperature (K)
t	normal distribution variable (–)
w	width of the focal image (m)

Greek symbols

α_{eff}	effective absorbance of the cavity
ε	subtended angle of the sun
ε^*	emissivity
σ	standard deviation (mrad)
σ^*	Stefan–Boltzmann constant
η	efficiency (–)
ρ	surface reflectance (–)
φ	intercept factor (–)
θ	inclination angle of the cavity (°)
υ	wind speed (m/s)
ψ	rim angle (°)

Subscripts

abs	absorber
amb	ambient
ap	aperture
ins	insulator
cond	conduction
conv	convection
cav	cavity
for	forced
Eff	effective
ext	outside
int	inside
rad	radiation
ref	reflector
rec	receiver
nat	natural
tub	pipe

optimise geometrical parameters for Stirling engines based on a theoretical analysis. Nepveu et al. [13] presented a model for a parabolic dish concentrator, known as a Eurodish, coupled to a 10 kW Stirling engine.

Halit et al. [14] presented an experimental study on the development of a beta-type Stirling engine for low and moderate temperature heat. Fraser [15] presented a model to estimate the performance of an Alpha-type Stirling engine, and the energy that is transmitted to the receiver is calculated by the equation proposed by Duffie and Beckman [16].

This equation states that the energy concentrated in the receiver of a solar concentrator is directly proportional to the direct radiation, the aperture opening area of the reflector, reflectivity and intercept factor. The latter concept is particularly useful in establishing the dimensions of the concentrator because reducing the receiver aperture opening diameter as a percentage of the solar image and incorporating the thermal model of the receptor defines the dimensions of the receiver for minimum heat loss.

There are different models available to estimate the intercept factor; Jaffe [17] and Badescu [18] presented a model that involves optical, thermal and conversion aspects of energy to optimise the dimensions of the parabolic dish concentrator. Romero [19] developed software for Sandia Laboratories that determined the energy that is intercepted by the circular or rectangular segments of a parabolic dish, which involves the optical aspects of the concentrator. Fraser [15] and Badescu [20] provided the results that are the most appropriate for parabolic dish concentrators that are coupled to a Stirling engine for different capacities. The models proposed by Badescu [21], Stine and Harrigan [22] considered both the influence of varying the receiver aperture opening diameter, which depends on the rim angle and height of the focal point, as well as the standard deviation of the errors caused by the geometrical factors and the optical concentration system.

After reviewing the literature, it was determined that no other previous model can be applied directly to evaluate the performance of an STSC because these models do not incorporate the geometric model of the STSC; therefore, in this study, a new mathematical model for an STSC coupled to a cavity receiver of a Stirling engine is presented. An analysis of the performance is carried out to compare an STSC and a parabolic dish to determine the technical viability of using this technology.

1.1. Description of an STSC

An STSC is the result of the interception of the open circular area with a small side section of a parabola, and this directs the solar radiation to a fixed focal point where a Stirling engine is installed and secured by a fixed support structure. It used a 2-axis tracking system, which consists of a daily tracking mechanism that moves the reflector mounted on a carriage in proportion to solar and other time-tracking mechanisms, which provides for rotation of the reflector that is synchronised with the movement of the sun during the day. This angular movement of the reflector is made around an axis oriented to maintain a fixed focal point normal to the incidence of the aperture opening area of the reflector, thus concentrating the solar radiation in the cavity that receives and heats a gas (helium, hydrogen or air) at high temperatures [23]. Later, the Stirling engine converts the heat into electricity, as observed in Fig. 1.

2. Materials and methods

2.1. Mathematical model

In this section, the mathematical model for attaching an STSC to a Stirling engine is developed using the mathematical estimation of