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Lattice Boltzmann simulation of natural convection in an open ended cavity

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ABSTRACT

Natural convection in an open ended cavity is simulated using Lattice Boltzmann Method (LBM). The paper is intended to address the physics of flow and heat transfer in open end cavities and close end slots. The flow is induced into the cavity by buoyancy force due to a heated vertical wall. Also, the paper demonstrated that open boundary conditions used at the opening of the cavity is reliable, where the predicted results are similar to conventional CFD method (finite volume method, FVM) predictions. Prandtl number (Pr) is fixed to 0.71 (air) while Rayleigh number (Ra) and aspect ratio (A) of the cavity are changed in the range of 10^4–10^6 and of 0.5–10, respectively. It is found that the rate of heat transfer deceases asymptotically as the aspect ratio increases and may reach conduction limit for large aspect ratio. The flow evaluation in the cavity starts with recirculation inside the cavity, as the time proceeds the flow inside the cavity communicates with the ambient.

1. Introduction

There is no need to say that Lattice Boltzmann Methods (LBM) are in high pace development and have become a powerful method for simulation fluid flow and transport problems for single and multiphase flows [1,2]. In this work, the method is applied for natural convection in open cavities. Natural convection in open cavies and slots are encountered in many engineering applications, such as solar thermal receiver, heat convection from extended surfaces in heat exchangers, solar collectors with insulated strips [3], etc. Few numerical simulations in open cavities were reported for aspect ratio of unity without inclinations [4–6] and with inclinations [7,8]. On the other hand few research papers have been published on experimental studies of buoyant flow in open cavities [9–11]. Effect of conduction (conjugate effect) long the boundaries of the cavities and radiative heat transfer on the heat transfer were addressed by [12–14]. Stability of flow in open cavities exposed to stratified media is addressed by [15]. It is found that homogenous flow is steady for the range of investigated parameters ($Ra = 5 \times 10^6 – 1 \times 10^{10}$, $Pr = 0.7$). However, for a high Ra, the stratified flow exhibits low and high frequency signals of the same types as in a closed cavity flow. The mechanisms of those frequencies were identified.

Most of the mentioned works investigated natural convection in cavities of aspect ratio of unity. The effect of systematic analysis of aspect ratio on the physics of flow and heat transfer is missing from the literature, which is worth being investigated. The velocity field and temperature profile are unknown at the opening boundary prior to solution. Such a boundary condition has never been tested for LBM applications before, which will be addressed in the present work. First the predictions of LBM are compared with predictions of finite volume method. The effect of aspect ratio on the flow and heat transfer systematically investigated. Also, the flow evaluation in the cavity is discussed.

2. Method of solution

Standard D2Q9 for flow and D2Q4 for temperature, LBM method is used in this work [1]; hence only brief discussion will be given in the following paragraphs, for completeness.

The BGK approximation lattice Boltzmann equation without external forces can be written as,

$$f_i(\mathbf{X} + \mathbf{c_i}\Delta t, t + \Delta t) - f_i(\mathbf{X}, t) = \Omega_i \tag{1}$$

where f_i are the particle distribution is defined for the finite set of the discrete particle velocity vectors $\mathbf{c_i}$. The collision operator, Ω_i, on the right hand side of Eq. (1) uses the so called Bhatangar-Gross-Krook (BGK) approximation [2]. For single time relaxation, the collision term Ω_i will be replaced by:

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$$\Omega_i = -\frac{f_i - f_i^{eq}}{\tau} \tag{2}$$

where τ ($\tau = 1/\omega_m$) is the relaxation time and f_i^{eq} is the local equilibrium distribution function that has an appropriately prescribed functional dependence on the local hydrodynamic properties.

The equilibrium distribution can be formulated as [2]:

$$f_i^{eq} = \omega_i \rho \left[1 + 3\frac{\mathbf{c_i}\cdot\mathbf{u}}{c^2} + \frac{9}{2}\frac{(\mathbf{c_i}\cdot\mathbf{u})^2}{c^4} - \frac{3}{2}\frac{\mathbf{u}\cdot\mathbf{u}}{c^2}\right] \tag{3}$$

where $\mathbf{u}$ and ρ are the macroscopic velocity and density, respectively, and ω_i are the constant factors, for D2Q9 is given as,

$$\omega_i = \begin{cases} 4/9 & i = 0,\ \textit{rest particle} \\ 1/9 & i = 1,3,5,7 \\ 1/36 & i = 2,4,6,8 \end{cases}. \tag{4}$$

The discrete velocities, $\mathbf{c_i}$, for the D2Q9 (Fig. 1) are defined as follows:

$$\begin{aligned} &\mathbf{c}_0 = (0,0) \quad \mathbf{c}_k = c(\cos\theta_k, \sin\theta_k) \quad \theta_k = (k-1)\pi/2 \\ &\textit{for} \quad k = 1,2,3,4\mathbf{c}_k = c\sqrt{2}(\cos\theta_k, \sin\theta_k) \quad \theta_k \\ &\quad = (k-5)\pi/2 + \pi/4 \quad \textit{for} \quad k = 5,6,7,8 \end{aligned} \tag{5}$$

where $c = \Delta x/\Delta t$, Δx and Δt are the lattice space and the lattice time step size, respectively, which are set to unity. The basic hydrodynamic quantities, such as density ρ and velocity $\mathbf{u}$, are obtained through moment summations in the velocity space:

$$\rho(\mathbf{X},t) = \sum_i f_i(\mathbf{X},t) \tag{6}$$

$$\rho\mathbf{u}(\mathbf{X},t) = \sum_i \mathbf{c_i} f_i(\mathbf{X},t) \tag{7}$$

The macroscopic viscosity is determined by

$$\nu = \left[\tau - \frac{1}{2}\right] c_s^2 \Delta t \tag{8}$$

Where c_s is speed of sound and equal to $c/\sqrt{3}$

For scalar function (temperature), another distribution is defined,

$$g_k(x + \Delta x, t + \Delta t) = g_k(x,t)[1 - \omega_s] + \omega_s g_k^{eq}(x,t) \tag{9}$$

D2Q4 is used to model transport of heat, for details see reference [1].

The equilibrium distribution function can be written as,

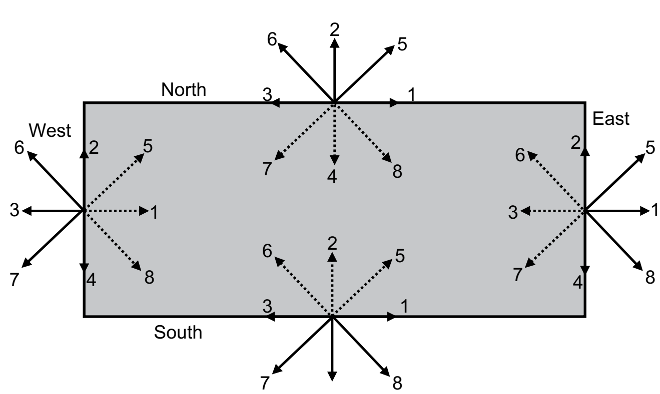

Fig. 1. Domain boundaries and direction of streaming velocities.

$$g_k^{eq} = w_k \phi(x,t)\left[1 + \frac{c_k \cdot u}{c_s^2}\right] \tag{10}$$

Notice that ω is different for momentum and scalar equations.

For momentum,

$$\omega_m = \frac{1}{3\cdot\nu + 0.5}, \tag{11}$$

where ν is the kinematic viscosity

and for the scalar

$$\omega_s = \frac{1}{2\cdot\alpha + 0.5}, \tag{12}$$

where α is the diffusion coefficient (thermal diffusion coefficient).

Nusselt number is calculated as,

$$Nu = -\frac{\partial T}{\partial Y} \tag{13}$$

Nusselt number is based on the height of the cavity, H. T stands for dimensionless temperature. Average Nusselt number is calculated by integrating eq. (13) along the height of the cavity and dividing by number of lattices along the height.

The standard LBM consists of two steps, streaming and collision. D2Q9 is used to solve the velocity field and D2Q4 is used to solve for the temperature field. The number of lattices used in x- and y-direction depends on the aspect ratio. However, at least 100 lattices are used in y-direction and number of lattice in x-direction is aspect ratio multiplied by the number of lattices in the y-direction. The buoyancy force term is added as an extra source term to equation (1), as,

$$F_b = 3w_k g\beta\Delta T \tag{14}$$

where g, β and ΔT are gravitational acceleration, thermal expansion coefficient and temperature difference.

3. Boundary conditions

The distribution functions out of the domain are known from the streaming process. The unknown distribution functions are those toward the domain. Fig. 1 shows the unknown distribution function, which needs to be determined, as dotted lines.

Flow:

Bounce back boundary condition is used on the solid boundaries (west, north and south boundaries). At the east open boundary, the following condition is used,

$$f_{6,n} = f_{6,n-1}, \quad f_{3,n} = f_{3,n-1} \ \textit{and}\ f_{7,n} = f_{7,n-1} \tag{15}$$

Where n is the lattice on the boundary and $n-1$, is the lattice inside the cavity adjacent to the boundary.

Temperature:

Bounce back boundary condition (adiabatic) is used on the north and south of the boundaries. Temperature at the west wall is know, $T_w = 1.0$. Since we are using D2Q4, the only unknown is g_1, which is evaluated as,

$$g_1 = 0.5\,T_w - g_3 \tag{16}$$

The boundary condition for east wall needs special treatment, since prior to solution, the advected velocity direction is not known. It is assumed that if the flow is penetrating into the cavity, then the temperature should be ambient, $T = 0$, and if the flow leaving the