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Use of the kriging method in determining the properties of gases in large channels

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A B S T R A C T

Properties of fluids can vary significantly over a cross-section of a channel. The variations can be graphically presented by means of profiles which are typically based on a limited number of measured points. The article presents suitability of the kriging interpolation method for data analysis at measuring properties of fluids in large channels in various thermal and process plants. Using a practical example rather than complex statistical analyses the advantages of the kriging over other interpolation methods are presented. Several examples also give some guidelines on choosing number and distribution of measuring points to ensure accurate profiles.

1. Introduction

Measurement of the parameters of gases or liquids in power plants is important for monitoring the operation of the entire plant, both from the viewpoint of energy efficiency and reliability of operation, and regarding its influence on the environment. In channels and pipes of larger dimensions, the values of a certain parameter may be quite non-uniform over the channel's cross-section. Regardless of whether only the average value of the parameter is important or the parameter's variation over the cross-section is investigated, in such cases the value of the parameter needs to be measured in a bigger number of points [1,2]. Because of the way measurements are performed, the number of measurement points is limited, therefore even grid measurements cannot provide data on the value of the measured parameter at any particular point of the channel's cross-section. However, on the basis of the measured values, it is certainly possible to estimate the value of the chosen parameter in places where measurement was not done. Several interpolation methods have been developed for this purpose. While linear interpolation is the least complex interpolation method it does require appropriate tessellation of the area where calculations are to be performed. For planar cases Delaunay triangulation is commonly used [3,4]. Polynomial regression can also be used to find values of the chosen parameter where measurement was not or could not be performed [5]. In the field of geostatistics, the kriging method is often used, in which the unknown value of a parameter is calculated as the weighted average of known, measured values [6–9]. In this manner it is possible to determine the value of a studied parameter at any chosen number of points within a certain channel cross-section. By calculating parameter values in a sufficiently large number of points over the entire cross-section, the so-called profile of parameter variation across the entire cross-section is also obtained. All the mentioned methods have advantages and disadvantages regarding the particular case where they are applied. Linear interpolation and kriging retain values of the observed parameter in the points where the values are measured and they can be assumed to be correct. Polynomial regression on the other hand causes some 'smoothing' of the profile depending on the polynomial function used and the actual profile of the parameter. Linear interpolation and kriging perform best within the region of the available data while polynomial regression can easily be extrapolated beyond the region of the measurement points.

This paper briefly presents the procedure for planar interpolation using the kriging method, along with its use on an example of determination of flue gas properties over channel cross-section behind a rotational air heater. The conditions within the channel are estimated by numerical simulation. A combined CFD and regenerative heat transfer model was used to simulate three-dimensional velocity, pressure, temperature and gas composition fields within a rotary air heater and the adjoining flue gas channels [10,11]. The actual profiles are therefore assumed to be thoroughly

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Nomenclature

d	distance
i, j	integer counter
K	constant
l	lag distance
N	number of data points
n	number of point pairs within lag
P	computational point
T	data points
w	weight
$\mathbf{w}$	vector of weights
x,y,z	Cartesian coordinates
γ	general function
ϕ	general parameter

known, and the results obtained using the kriging method can be compared with them.

2. Observed planar profile

For illustrative presentation of characteristics of determination of planar parameter profiles using kriging and polynomial regression method, a case of temperature profile of flue gases in a channel with sides of 8 m and 3.6 m that is located behind a rotational heat exchanger was used. The temperature profile was calculated by means of numerical simulation of the operation of rotational heat exchanger. Fluid flow simulations are based on solving a system of transport equations which is done with commercial CFD software. Additional model was used to simulate regenerative heat transfer within the matrix of the heat exchanger [10,11]. The simulation yields the values of flue gas temperature throughout entire three-dimensional computational domain including the studied plane. The virtual measuring plane thus consists of almost 4800 points where the observed parameter is known. Fig. 1 depicts the reference temperature profile on the measuring plane. This data will be used for comparison with the values calculated with spatial interpolation. Measurements within the channel are substituted with sampling a certain number of points on the virtual measuring plane; these represent known points T_i. The temperatures at the selected points thus represent known values of parameter ϕ_i. All profiles which will be presented in subsequent sections are shown in a comparable manner and using the same temperature scale as in Fig. 1.

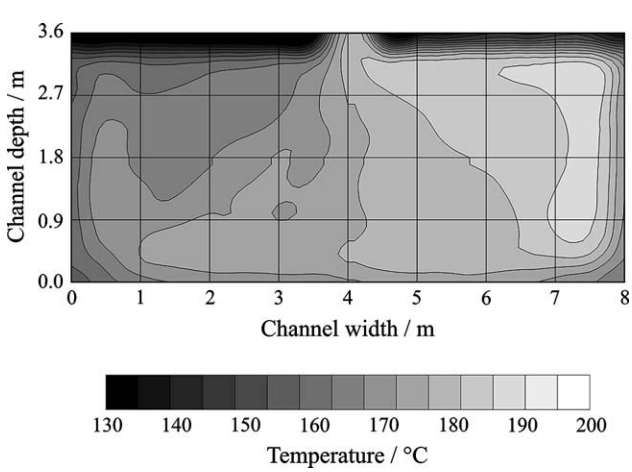

Fig. 1. Actual profile of the studied parameter – flue gas temperatures behind a rotational heat exchanger.

3. Spatial interpolation using the kriging method

In engineering practice and research, parameter measurements are often required. The values of such parameters may continuously vary within a certain space or along a plane. Measurements are usually performed in discrete points and the number of measurement points is limited by the available equipment and time. Furthermore, the locations of measurement points usually cannot be optional, because they are determined by the design of duct and accessibility of the area in which measurements are done. Even in places where individual parameter values are not known for any reason, these can be determined on the basis of known values at other points. For this purpose, an interpolation method is necessary that will take into account as many known points as possible and will therefore include the characteristics of variation of the studied parameter across the entire measurement range.

In the fifties of the previous century, the South African mining engineer Danie G. Krige studied a similar problem. On the basis of a limited number of soil samples, he tried to determine the content of a certain ore at sites from which he had no available samples. French mathematician Georges Matheron further developed the interpolation method proposed by Krige and named it kriging [6,8]. Nowadays, this method is used mostly in geostatistics, but it is also suitable for the analysis of measurements in various fields of engineering, for example in power and process plants [2,12,13].

In the case of the kriging method, estimation of the value of any chosen parameter $\phi = \phi(x,y,z)$ in a computational point $P = (x,y,z)$, where its value is unknown, is based on the known values ϕ_i of the same parameter in N data points $T_i = (x_i,y_i,z_i)$ in the vicinity of point P. The estimate of the sought value is the weighted average of all known values [6,8].

$$\phi = \sum_{i=1}^{N} w_i \phi_i \tag{1}$$

Factors w_i are weights which represent the influence of individual data point T_i on the value of parameter ϕ at point P. All weights together compose the vector of weights $\mathbf{w}$ for point P. The sum of vector components i.e. all weights w_i should equal 1 to guarantee uniform unbiasedness of the estimated values [8], i.e. the average value of the actual parameter values is the same as the average value of the estimated parameter values [6].

$$\sum_{i=1}^{N} w_i = 1 \tag{2}$$

3.1. Weights for calculation of parameter values in computational points

The values of weights for individual points are determined on the basis of the assumption that an unknown value of parameter ϕ at point P is more likely to be similar to the values at points T_i close by than in more remote points. The weights of points which lie closer to the computational point P will therefore be higher, while those for more remote points will be lower, possibly even negligibly small [6]. The influence of distance can generally be described using appropriate function $\gamma(d_{i,j})$ in which $d_{i,j}$ represents the distance between points i and j. The optimal weights are calculated using the following system of linear equations: