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Mixed convection boundary layer flow past an isothermal horizontal circular cylinder with temperature-dependent viscosity

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ABSTRACT

The problem of steady laminar mixed convection boundary layer flow past an isothermal horizontal circular cylinder placed in a viscous and incompressible fluid of temperature-dependent viscosity is theoretically considered in this paper. The partial differential equations governing the flow and heat transfer are shown to be non-similar. Full numerical solutions of these governing equations are obtained using an implicit finite-difference scheme known as the Keller-box method. The solutions are obtained for various values of the Prandtl number Pr, the mixed convection parameter λ and the viscosity/temperature parameter θ_r. The obtained results show that the flow and heat transfer characteristics are significantly influenced by these parameters.

1. Introduction

Mixed or combined convective heat transfer in boundary layer flow represents an important problem, which is frequently encountered in many industrial and technical processes including solar central receivers exposed to winds, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchangers placed in a low-velocity environment, etc. [1]. Further, the typical physical example of variable fluid properties can be found in the oil cooling of the electronic equipment [2]. In this kind of convective flows, the free and forced convection effects are of comparable magnitude [3]. It is very well-known that in several practical applications, there exist significant temperature differences between the surface of the hot body and the free stream. These temperature differences cause density gradients in the fluid medium and in the presence of a gravitational body force, free convection effects become important. When natural or mixed convection heat transfer takes place under conditions where there are large temperature differences within the fluid, it is necessary (for accuracy) to consider the effects of variable fluid properties. The typical physical example can be found in the oil cooling of the electronic equipment.

Studies on mixed convection boundary layer flow past a horizontal circular cylinder have been conducted previously by several researchers. It appears that Sparrow and Lee [4] were the first to study the problem of mixed convection boundary layer flow about a horizontal circular cylinder. Later, Merkin [5] has studied the problem of mixed convection from a horizontal circular cylinder and he solved it based on the Crank–Nicolson method, using Newton–Raphson method along with Choleski decomposition technique. Further, Nazar et al. [6–9] have solved similar problems for Newtonian as well as for micropolar fluids and they have considered the cases of constant wall temperature and constant wall heat flux.

An interesting macroscopic physical phenomenon in fluid mechanics is the variation of viscosity with temperature. For many liquids, among them petroleum oils, glycerine, silicone fluid and some molten salts, the variation of absolute viscosity with temperature is often much greater than that of the other properties [2]. In all of the above mentioned studies on mixed convection boundary layer flow past a horizontal circular cylinder, the fluid viscosity is treated as a constant. However, it is well-known that the changes of this physical property may relate to the temperature. For example, the viscosity of water decreases by about 58% when the temperature increases from 10 °C ($\mu = 1.31 \times 10^{-3}$ kg m^{-1}s^{-1}) to 50 °C ($\mu = 5.48 \times 10^{-4}$ kg m^{-1}s^{-1}). In order to make good predictions of the flow behaviour, it is important to take into account this variation of viscosity. It has been shown by Gary et al. [10] and

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Nomenclature

a	radius of the cylinder
C_f	skin friction coefficient
f	non-dimensional stream function
g	acceleration due to gravity
Gr	Grashof number, $Gr = g\beta\Delta Ta^3/\nu^2$
k	thermal conductivity
Nu	Nusselt number
Pr	Prandtl number
q_w	heat flux from the cylinder
Re	Reynolds number, $Re = U_\infty a/\nu$
T	local fluid temperature
T_r	reference temperature
T_w	cylinder temperature
T_∞	ambient temperature
au, v	non-dimensional velocity components along the x – and y – directions, respectively
$u_e(x)$	non-dimensional velocity outside boundary layer
U_∞	free stream velocity
x, y	non-dimensional Cartesian coordinates along the surface of the cylinder and normal to it, respectively
x_s	boundary layer separation point

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
γ	thermal property of the fluid
θ	non-dimensional temperature
θ_r	viscosity/temperature parameter
λ	mixed convection parameter
ν	kinematic viscosity, $\nu = \mu/\rho$
ν_∞	constant kinematic viscosity of the ambient fluid
μ	dynamic viscosity
μ_∞	constant dynamic viscosity of the ambient fluid
ρ	fluid density
τ_w	wall shear stress
ψ	stream function

Subscripts

w	condition at the surface of the cylinder
∞	ambient/free stream condition

Superscripts

$'$	differentiation with respect to y
$-$	dimensional variables

Mehta and Sood [11] that when variable viscosity is taken into account, the flow characteristics may be substantially changed compared to the constant viscosity case. Lings and Dybbs [12] studied the forced convection with variable viscosity over flat plate in a porous medium. Kafoussias and Williams [1] and Kafoussias et al. [13] have investigated the effect of temperature-dependent viscosity on mixed convection boundary layer flow past a vertical isothermal flat plate. Hossain et al. [14,15] studied the natural or free convection flow about a vertical cone and vertical wavy surfaces, respectively, with the viscosity inversely proportional to the linear function of temperature. Further, Molla et al. [16] have considered a problem of natural convection flow about an isothermal horizontal circular cylinder with temperature-dependent viscosity. Recently, Ahmad et al. [17] have studied the effect of temperature-dependent viscosity on free convection over isothermal cylinders of elliptic cross section. Finally, we mention to this end the interesting paper by Ali [18] where he considered the problem of the effect of variable viscosity on mixed convection heat transfer along a vertical moving surface.

Therefore, in order to get more accurate information about the flow and temperature characteristics, the aim of this paper is to study the problem of steady laminar mixed convection boundary layer flow past an isothermal horizontal circular cylinder with the effect of temperature-dependent viscosity immersed in a viscous fluid for both assisting (heated cylinder) and opposing flow (cooled cylinder) cases. The partial and ordinary differential equations governing the flow and temperature fields are solved numerically using an efficient implicit finite-difference scheme known as the Keller-box method (see [19]). The results obtained are compared with those reported by Merkin [5] for a constant viscosity when the Prandtl number is unity, namely $Pr = 1$ and it is found that the results are in very good agreement. It should be mentioned that the method used by Merkin [5] consists in replacing derivatives of the partial differential equations in the direction by differences and all other quantities averaged. The two non-linear ordinary differential equations which result were solved by writing them in finite-difference form and solving the non-linear algebraic equations iteratively by a Newton–Raphson process. The linear algebraic equations arising in the iterative process were solved by Choleski decomposition technique. We believe that the present results are more accurate than those which use the usual assumption of constant properties [20].

2. Mathematical formulation

We consider a problem of mixed convection boundary layer flow past a horizontal circular cylinder of radius a placed in a viscous and incompressible fluid of temperature-dependent viscosity. It is also assumed that the cylinder is kept at the uniform temperature T_w, while the ambient fluid has the constant temperature T_∞, where $T_w > T_\infty$ (heated cylinder) corresponds to an assisting flow (free stream and buoyancy forces are in the same direction where the buoyancy forces will assist the fluid to accelerate in the boundary layer) and $T_w < T_\infty$ (cooled cylinder) corresponds to an opposing flow (free stream and buoyancy forces are in the opposite directions where the buoyancy forces will retard the fluid flows in the boundary layer). Further, following Merkin [5] it is assumed that the characteristic velocity is $(1/2)U_\infty$. Under these assumptions along with the Boussinesq and boundary layer approximations, the basic boundary layer equations of this problem are:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \rho \bar{u}_e(\bar{x})\frac{d\bar{u}_e(\bar{x})}{d\bar{x}} + \frac{\partial}{\partial \bar{y}}\left(\mu \frac{\partial \bar{u}}{\partial \bar{y}}\right) + \rho g\beta(T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right), \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

subject to the boundary conditions

$$\bar{u} = \bar{v} = 0,\ T = T_w \text{ at } \bar{y} = 0, \quad \bar{u} \to \bar{u}_e(\bar{x}),\ T \to T_\infty \text{ as } \bar{y} \to \infty, \tag{4}$$

where $\bar{x}$ is the coordinate measured along the surface of the cylinder starting from the lower stagnation point ($\bar{x} \approx 0$) and $\bar{y}$ is the