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Review

A critical review of buoyancy-induced flow transitions in horizontal annuli

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ABSTRACT

The main mechanisms of transition of buoyancy-induced flows in the horizontal annulus between circular cylinders are reviewed, based on the available literature. Both experimental and theoretical studies are considered. The different scenarios for the evolution of the flow regimes and temperature patterns are tracked, for increasing values of the Rayleigh number, *Ra*. The occurrence of various instability and bifurcative phenomena is pointed out, and linked to other relevant parameters, such as the radius ratio *R* and the Prandtl number, *Pr*. Although most of the relevant literature is on 2D cases, the effect of the third dimension is considered as far as possible. Studies on the influence of the eccentricity of the inner cylinder on the laminar flow and the thermal asset are also reviewed. Finally, open questions and topics for future research are hinted at.

1. Introduction

Buoyancy-induced flows in enclosures may be very complex in nature, and highly unpredictable in terms of thermal effectiveness. This is partially inherent in the bi-directional interaction between the flow and temperature fields, but also derives from the sensitivity of the thermal-flow regimes to the geometric and thermal configuration of the system.

Heat transfer by natural convection is characterised by relatively low heat transfer rates, and a possible way for enhancement is the exploitation of transitional regimes. These are time-varying in nature, and could therefore be much more efficient than steady-state regimes in terms of heat transfer performance. The prediction and control of transitional buoyant flows are however increasingly difficult as the Rayleigh number increases, due to the wide spectrum of flows potentially arising from successive bifurcations.

The importance of buoyancy-induced flow transitions as a research topic goes far beyond the field of thermal sciences. In fact, it is deeply entwined with the history of chaos theory, since meteorologist Edward N. Lorenz [1] witnessed for the first time the occurrence of deterministic, but nevertheless unpredictable behaviour arising from the numerical analysis of a dynamical system. Such a system represented a simplified model of the flow within an infinite layer of fluid heated from below, and consisted of three equations, representing a truncation of a spectral expansion of the equations of natural convection. His analysis proved that complex chaotic dynamics could be reproduced even by a low-dimensional system, and later observations [2] confirmed that analogous dynamics can be recovered in the study of real cases, even though the Lorenz model is not at all representative of a real problem.

So far, most of the research effort in this field has been dedicated to the study of buoyant flow instabilities and bifurcations in basic geometries. Rayleigh–Bénard convection in a horizontal layer of fluid heated from below and the side-heated enclosure with rectangular cross section and adiabatic top and bottom walls are the most widely studied cases. Some of the main results concerning these cases are summarized in a number of review papers, among which those of Yang [2] and Le Quéré [3] are very noteworthy.

A third fundamental case of confined free convection is the flow originating in the region between two horizontal cylinders with differential heating. The case is commonly referred to as the horizontal annulus. In the last decades, a considerable amount of work has been published on the stability and bifurcations of natural convection flows in horizontal annuli, but a comprehensive review of the results achieved so far is still lacking. This survey intends to partially fill this gap, by providing an outlook on the mechanisms of transition, in order to sketch the guidelines for developing future research.

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Nomenclature

a	wave number
A	aspect ratio
$A_y = L_y/H$	axial aspect ratio
e	eccentricity [m]
g	gravitational acceleration [m/s^2]
$\widehat{\mathbf{g}}$	gravity unit vector
$Gr = g\beta\Delta TL_{ref}^3/\nu^2$	Grashof number
h	convective heat transfer coefficient [W/(m^2 K)]
H	gap width [m]
k	thermal conductivity [W/(m K)]
L_y	axial length [m]
$Nu = hL_{ref}/k$	Nusselt number
P	dimensionless piezometric pressure
$Pr = \nu/\alpha$	Prandtl number
r	radius [m]
$R = r_o/r_i$	radius ratio
$Ra = GrPr$	Rayleigh number
t	dimensionless time
T	dimensionless temperature
ΔT	driving temperature difference $(T_H - T_C)$ [K]
$\mathbf{u}$	dimensionless velocity vector

Greek symbols

α	thermal diffusivity [m^2/s]
β	thermal expansion coefficient [1/K]
δF	Feigenbaum constant
$\varepsilon = e/H$	dimensionless eccentricity
μ	general leading parameter of a dynamical system
ν	kinematic viscosity [m^2/s]
φ	eccentricity angle
θ	angular coordinate

Subscripts

c	critical
C	cold
H	hot, based on height
i	inner
o	outer, oscillatory
ref	reference

2. Theoretical background

Under the Boussinesq approximation, the dimensionless governing equations for natural convection read as follows:

$$\nabla\cdot\mathbf{u} = 0 \tag{1}$$

$$\frac{\partial\mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla P + \frac{1}{Gr^{\frac{1}{2}}}\nabla^2\mathbf{u} - \widehat{\mathbf{g}}T \tag{2}$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = \frac{1}{Gr^{\frac{1}{2}}Pr}\nabla^2 T \tag{3}$$

From the point of view of mathematical physics, the system (1)–(3) represents an autonomous, non-linear, dissipative, dynamical system, whose trajectories belong to an ∞-dimensional space. The control parameter of the system is the Grashof number, Gr, or, equivalently, the Rayleigh number $Ra = GrPr$. Other parameters that influence the system behaviour are the Prandtl number Pr, the geometrical ratios of the domain, and the thermal boundary conditions.

When reviewing works on confined buoyant flows, several types of flow regimes are encountered, of which a qualitative classification can be given. It is useful to introduce a simple set of acronyms and abbreviations, in order to facilitate the description of such flows and the explanation of the global patterns of transition associated with them. The notation by Gollub and Benson [4] is followed and properly extended where necessary [5]:

PD indicates a pseudo-diffusive flow [6,7]. Pseudo-diffusive flow regimes consist of steady state, very weak, shear-driven circulations, induced by a diffusion-dominated temperature field.

S denotes a generic steady-state convective flow, normally organized in a number of circulation cells.

P denotes a time-dependent, periodic flow, represented by a closed orbit in phase space, and typically originated by a Hopf bifurcation of a *S*-type flow.

P_n indicates another periodic state, emerging after a series of period doublings of a *P*-type flow. Its corresponding attractor is again a closed orbit.

L indicates a periodic flow with a number of frequencies locked, by twos, to a rational ratio (phase locking). In this case, the flow regime is represented by a closed orbit on a torus.

QP_n denotes a quasi-periodic flow, characterised by an oscillation presenting n independent, incommensurable frequencies. In phase space, the trajectory associated with a quasi-periodic flow lies on the surface of a torus.

I denotes an intermittent flow regime, i.e. a regime characterised by alternating periodic phases and chaotic "bursts", whose representation in phase space is a strange attractor.

N indicates a non-periodic, chaotic flow, represented by a strange attractor as well.

Details of transition vary greatly from one case to another; however, a number of established routes to chaos are found to be common to many types of dissipative dynamical systems, whose validity is supported by a strong body of theory [8].

One of the most recurrent routes to chaos in enclosed natural convection is the Ruelle–Takens scenario. This scenario consists in the birth of a strange attractor through three Hopf bifurcations: the first one turns a fixed point into a periodic orbit; through the second one, the orbit bifurcates in a quasi-periodic flow; the third one sees the birth of a third incommensurate frequency. If the leading parameter is increased further, a strange attractor is likely to appear. According to the above classification, the Ruelle–Takens scenario can be represented by the following sequence: $S \to P \to QP_2 \to QP_3 \to N$.

Another common route to chaos is the so-called period-doubling or Feigenbaum route. In this scenario, a stable periodic orbit triggers an infinite sequence of period-doubling bifurcations, leading to chaos, which can be represented as: $S \to P \to P_2 \to P_4 \to \ldots \to N$. This cascade of period-doubling bifurcations is such that:

$$\lim_{k\to\infty}\frac{|\mu_k - \mu_{k-1}|}{|\mu_{k+1} - \mu_k|} = \delta_F = 4.6642016\ldots \tag{4}$$

where μ is the control parameter, and δ_F is the Feigenbaum constant. This is a universal constant, in that it can be detected on any occasion an infinite period-doubling cascade occurs.