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Calibration of constitutive equations under conditions of large strains and stress triaxiality

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ABSTRACT

Constitutive equations were calibrated to improve their application in assessing a stress field in front of a crack under the conditions of large strains and stress triaxiality. The Bai–Wierzbicki method was adopted, and certain changes and new terms were introduced to incorporate material softening. Five shapes of specimens were tested to cover a wide range of stress triaxiality conditions and Lode factors. Tests were performed at three different temperatures, namely, +20 °C, –20 °C, and –50 °C, and on three different materials obtained by three different heat treatments of S355JR steel.

1. Introduction

Classical phenomenological fracture mechanics provides several fracture criteria to predict the onset of crack propagation. These criteria are widely used for both brittle and ductile materials. When using these criteria, certain quantities are computed analytically or numerically (J – integral, J_I; stress intensity factor, K_I; energy release rate, G_I, or crack tip opening displacement, CTOD). They characterize the mechanical fields in front of the crack and at the critical moment they must be equal to their critical values. These values, in turn, must be determined experimentally according to certain national or international standards. These quantities characterize the fracture toughness and strongly depend on the size and shape of specimens or structural members. The fracture toughness is measured according to standards and provides conservative estimates of the critical loading conditions. Several research results have introduced corrections to classical fracture toughness parameters measured according to such standards, e.g., [1–4]. These corrections reduce the conservative nature and simultaneously provide more economical estimations. However, another approach and philosophy can be used in predicting the fracture of structural elements. This approach is called the local approach, and Beremin [5] is typically considered the initiator of a series of research efforts based on the associated concepts. The concept of Weibull stress and probability of fracture was developed by French group e.g., [6–9] (review publication), and by Dodds' group, e.g., [10–13]. Another approach, which focuses on the damage evolution of the material microstructure, is typically called the GTN model. This approach was initiated by Gurson [14] and extended by Chu and Needleman [15] to add the term concerning the void nucleation process, by Tvergaard and Needleman [16] to incorporate void growth and coalescence, by Leblond [17] to incorporate strain hardening, and by Pardoen and Hutchinson [18] to incorporate the void shape into the analysis. Benzerga et al. [19] extended the theory to accommodate plastic

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anisotropy, and Nahshon and Hutchinson [20], along with Nielsen and Tvergaard [21], introduced the Lode parameter. The third important group of research reports that furthered local analysis methods includes Bao and Wierzbicki's study [22] of the concept of the critical strain to fracture, the Mohr–Coulomb hypothesis and the invariants of the stress tensor and deviator of the stress tensor. Several reports have been published in this area [23–26]. In the methods developed in these three groups of studies, one must know the stress and strain distributions locally within the loaded element. This knowledge becomes essential when the damage evolution is discussed under the conditions of plastic deformation and large strains. Such a situation is met in front of the crack within an elastic–plastic material. The local theory of fracture incorporates the distribution of opening stress in front of the crack to compute the Weibull stress. In the GTN model, the process of void nucleation – growth – coalescence is directly taken into account using certain postulated functions, but the yield function must be properly calibrated. In the Bai–Wierzbicki formula to compute the critical strain, certain quantities must be computed by calibrating the yield function.

In this paper, we discuss the process of uniaxial stress–strain curve calibration in detail, for the strain range both before and after the maximum of the stress–strain curve. Since the accumulated effective plastic strains in front of the crack may reach the level of tens or hundreds of percent, a proper approximation and extrapolation of the true stress–logarithmic strain (TS–LS) curves is necessary, and additional corrections are required to adjust the numerical results to match the experimental data.

Here, we use the approach of Bai–Wierzbicki [24] with certain adjustments. Thus, the stress–strain curve contains the functions of invariants of the stress tensor and deviator of the stress tensor. The stress–strain curves are calibrated using five different specimen geometries and loading conditions and three materials tested at three temperatures.

2. General model

Bai and Wierzbicki postulated the following formula to compute the yield stress [24]:

$$\sigma_{yld} = \overline{\sigma}(\overline{\varepsilon}_p)\left[1-c_\eta(\eta-\eta_0)\right]\left[c_\theta^s + (c_\theta^{ax}-c_\theta^s)\left(\gamma-\frac{\gamma^{m+1}}{m+1}\right)\right] \quad (1)$$

This equation determines the shape of the yield surface, where $\overline{\sigma}(\overline{\varepsilon}_p)$ is the function between the effective stress and the effective accumulated plastic strain $\overline{\varepsilon}_p$; η is the triaxiality coefficient ($\eta = \sigma_m/\overline{\sigma}$, where $\sigma_m = 1/3(\sigma_{11}+\sigma_{22}+\sigma_{33}) = 1/3I$, I is the first invariant of the stress tensor, $\overline{\sigma} = \sqrt{3J_2}$ is the effective stress, and J_2 is the second invariant of the stress tensor deviator); η_0 is a reference value of the triaxiality coefficient and $\eta_0 = 1/3$ for the uniaxial tensile test. The γ function represents a curve drawn along the deviatoric surface between the contours defined by the Huber–von Mises and Tresca criteria in the principal stress space. The γ function satisfies the inequality $0 \le \gamma \le 1$, and $\gamma = 0$ for plane strain or pure shear, and $\gamma = 1$ for axial symmetry. Bai and Wierzbicki postulated that the γ function takes the following form:

$$\gamma = \frac{\cos(\pi/6)}{1-\cos(\pi/6)}\left[\frac{1}{\cos(\theta-\pi/6)}-1\right] = 6.464[\sec(\theta-\pi/6)-1] \quad (2)$$

where θ is the Lode angle, which is a function of the third invariant of the stress deviator,

$$\cos(3\theta) = \left(\frac{r}{\sigma_e}\right)^3 = \xi = \frac{27}{2}\frac{J_3}{\sigma_e^3} \quad (3)$$

$$r = \left[\frac{27}{2}det(s_{ij})\right]^{1/3} = \left[\frac{27}{2}(\sigma_1-\sigma_m)(\sigma_2-\sigma_m)(\sigma_3-\sigma_m)\right]^{1/3} \quad (4)$$

where s_{ij} is the stress tensor deviator. The Lode angle must satisfy the inequality $0 \le \theta \le \pi/3$. The normalized Lode angle $\overline{\theta}$ ($-1 \le \overline{\theta} \le 1$) is also used in the analysis.

$$\overline{\theta} = 1-\frac{6\theta}{\pi} = 1-\frac{2}{\pi}\arccos\xi \quad (5)$$

Another definition of the Lode parameter can be used:

$$L = -\frac{2\sigma_{II}-\sigma_I-\sigma_{III}}{\sigma_I-\sigma_{III}} \quad (6)$$

where σ_I and σ_{III} are the maximum and minimum principal stresses, respectively.

The following relationship exists between ξ and L:

$$\xi = L\frac{9-L^2}{\sqrt{(L^2+3)^3}} \quad (7)$$

The shape of the function (1) is not the only one proposed by Wierzbicki and co-workers. To our knowledge, no researchers have attempted to determine a universal function independent of specimen geometry. This problem is also discussed in the present paper. In Eq. (1), the quantity c_θ^{ax} is defined as follows:

$$c_\theta^{ax} = \begin{matrix} c_\theta^t & \text{for}\,\overline{\theta} \ge 0 \\ c_\theta^c & \text{for}\,\overline{\theta} < 0 \end{matrix} \quad (8)$$

Eq. (1) contains four parameters to be determined: c_θ^t, c_θ^c, c_θ^s and m. The term containing the m parameter is added to make the yield surface smooth and differentiable with respect to the Lode angle θ in the neighbourhood of $\gamma = 1$. These parameters must be determined experimentally. However, at least one of them is equal to unity. If $\overline{\sigma}(\overline{\varepsilon}_p)$ is found through a uniaxial tensile test using cylindrical specimens, then $c_\theta^t = 1$. If a uniaxial compression test is performed, then $c_\theta^c = 1$, and in the case of a shear test, $c_\theta^s = 1$. All four parameters can be selected in such a way that one obtains either the Huber–von Mises or Tresca yield surface.

The effective stress is computed as follows:

$$\overline{\sigma} = \sqrt{3J_2} = \sqrt{3/2 s_{ij}s_{ij}} = \\ = \left\{\frac{1}{2}\left[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{11}-\sigma_{33})^2 + (\sigma_{22}-\sigma_{33})^2\right] + 3(\sigma_{12}^2+\sigma_{13}^2+\sigma_{23}^2)\right\}^{0.5} \quad (9)$$

The plastic effective strain rate can be computed as follows: