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A new mixed-field theory for bending and vibration analysis of multi-layered composite plate

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ABSTRACT

A novel mixed-field theory with relatively low number of unknown variables is introduced for bending and vibration analysis of multi-layered composite plates. The presented plate theory is derived from a parametrized mixed variational principle which is introduced recently by the first author. A global-local kinematic with a layer-independent number of variables is assumed for the description of the displacements of the plate. The considered kinematic stratifies the displacement and transverse stress continuity conditions at the mutual interfaces of the layers. It also fulfill the homogenous boundary conditions of the shear stresses on the upper/lower surfaces of the plates without using the shear correction factor. One-unknown variable fields which satisfy a priori the continuity conditions at the adjacent interfaces between layers and the zero boundary conditions on the bounding surfaces are considered for the approximation of the transverse shear stresses. The transverse normal stress along the total thickness of the multi-layered plate is approximated via a quadratic polynomial. The presented mixed-field plate theory has been validated through comparison of the bending and vibration analysis results with those obtained from the three-dimensional (3D) theory of elasticity and the results of the other classical and high-order plate theories.

1. Introduction

Nowadays, multi-layered composite plates are used extensively in military, aeronautical, marine and mechanical engineering fields as primary structural elements. High-peculiar stiffness and strength, excellent corrosion resistance as well as the lightweight characteristic are some of the most reasons for the widespread use of composite structures in the industrial applications. The accurate prediction of the displacements and stresses in the multi-layered composites is essential for design purposes.

Various numerical and analytical approaches have been developed so far for the analysis of multi-layered composites structures. The mathematical approaches which are on the basis of finding a solution for the differential 3D equilibrium equations of a continuous media yields the most precise results for the multi-layered plates problems [1–5]. However, these approaches are restricted to the plates whose geometrical shapes, loading and boundary conditions are simple. At the

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first glance, it seems that the problems of the multi-layered plates for general case of arbitrary geometry, number of layers, stacking sequence, loading and boundary conditions can be solved using the 3D finite element (FE) analysis. However, 3D FE analysis of multi-layered structures is computationally cost and not possible in some practical conditions. The aforementioned difficulties inclined engineers to use the two-dimensional (2D) multi-layered plate theories. Available multi-layered plate theories can be classified into four categories: equivalent single layer theories; discrete layer or layer-wise (LW) theories; global-local theories; and mixed plate theories.

In the equivalent single-layer approaches, a global displacement field is assumed for all layers of the multi-layered structure. In this category, one can distinguish the classical layered theory (CLT) [6,7], first-order shear deformation theory (FSDT) [8–12] and high-order shear deformation theories (HSDT) [13–15]. Although the number of unknowns variables in the equivalent single-layer theories is independent of the number of material layers, the continuity conditions of transverse shear and normal stresses on the mutual interfaces of the layers are often violated.

In LW theories, a local kinematic is assumed for each material layer of the multi-layered plate. Plate theories based on LW fulfill the continuity conditions of out-of-plane stresses on the adjacent interfaces between layers. Due to the dependency of the number of kinematic variables to the number of layers, LW theories are computationally expensive [16–19].

In the global-local theories, a global kinematic describing the whole behavior of multi-layered plate is put on local layer kinematic selected based on the LW concepts. Through applying the stress and displacement continuity conditions on the mutual interfaces between the layers, the number of the kinematic unknown variables reduces significantly. Such models reproduce a continuous shear stress fields along the thickness direction of multi-layered plates [20–24].

Mixed plate theories consider two independent fields for displacement and stress components of the multi-layer plate. In addition to the displacement components, the stress components are also assumed as primary unknown field variables in these theories. Mixed based plate theories satisfy a priori the continuity conditions of the out-of-plane stresses along the layer interfaces and traction boundary conditions on the top/bottom surfaces [25–30].

The above reviewed literature deals with only some aspects of the broad research activity about multi-layered plate theories. An comprehensive assessment of different multi-layered theories has been done by Carrera [31].

The available mixed plate theories, which are mostly based on Reissner's mixed variational theorem (RMVT), consider a layer-wise polynomial scheme for the approximation of the out-of-plane stresses. Due to using the layer-wise scheme, the number of unknown field variables of these plate theories is very high and they are computationally very expensive. In this study, a new mixed-field plate theory with relatively low number of field variables are developed for the multi-layered composite plates. A layer-independent distribution with only one unknown variable is assumed for the approximation of the shear stress components. These assumed shear stress fields, which are obtained by integrating from 3D equilibrium equations, fulfill a priori the interfaces continuity conditions and the traction-free conditions on the top/bottom surfaces of the plate. Variations of the transverse normal stress field along the total thickness of the multi-layered plate is approximated via a second-order polynomial. The description of the displacement components of the multi-layered plate is also based on a new global-local kinematic. The employed kinematic has a constant (seven) number of unknown variables irrespective of the number of layers. Thanks to using 1,2-3 double superposition technique [32], the present global-local kinematic satisfies all continuity conditions of the shear stresses as well as compatibility conditions of the displacement components at the adjacent interfaces between the layers.

The governing equations of motion are numerically solved using a quadrilateral four-node plate element which has 21 degrees of freedom (dofs) at each node. In the FE formulation, the full Hermitian shape functions with C^1-continuity are employed for the interpolation of the transverse displacement of the multi-layered plate. Other displacement and stress field variables of the multi-layered plate have been interpolated via Lagrange shape functions with C^0-continuity.

The paper is outlined as follows: the descriptions of the proposed global-local displacement field and the out-of-plane stress fields are given in Section 2. The FE formulations of the four-node quadrilateral multi-layered plate are fully described in Section 3. Numerical results which include various bending and free vibration tests are subsequently presented in Section 4. Finally, conclusion and recommendations for future researches are cited in Section 5.

2. Theoretical formulation

2.1. Basic assumption

We consider a multi-layered composite plate of constant thickness h, consisting of N_l orthotropic layers (see Fig. 1). As shown in Fig. 1, the global Cartesian coordinate system (x,y,z) is located at the mid-plane (z = 0) of the multi-layered plate. At the mid-plane of each layer, an individual local coordinate system is also considered.

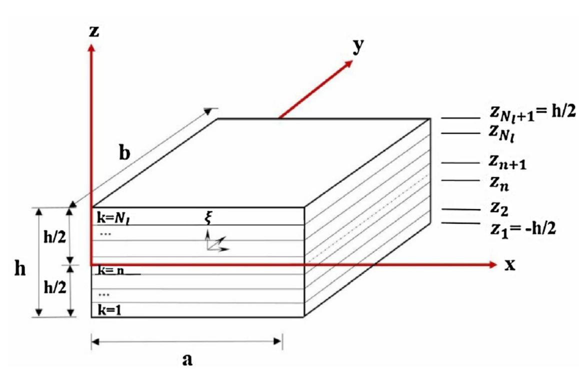

Fig. 1 – The multi-layered composite plate and the considered Cartesian coordinate system.