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Original Research Article

An input–output damage detection method using static equivalent formulation of dynamic vibration

Mohsen Mousavi [a], Amir H. Gandomi [b,*]

[a] Civil Engineering Department, University of Guilan, Rasht, P.O. Box 3756, Iran

[b] Stevens Institute of Technology, Hoboken, NJ 07030, United States

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ABSTRACT

In this study, a new damage detection method is developed which directly uses input–output data of a forced vibration of a structure. For this, the dynamic vibration formulation of an FE model has been integrated within the time domain of the vibration of the structure. Also, the static condensation scheme is used to reduce required measured degrees of freedom (DOF's). Hence, the main characteristic of the proposed method is that it just uses translational time history response of a structure at specified nodes corresponding to the finite element model of that structure. Also, the only required data from the original FE model of the structure is its stiffness matrix. To assess the capability of the proposed method in damage detection in beam type structures a cantilever beam is studied. Not only can the method locate damaged elements, but also the quantity of damage in every damaged element is computed successfully. Also, it has been shown that as the frequency of the applied load in simulated experiment approaches to the first natural frequency of the beam, the accuracy dwindles significantly. Hence, for obtaining more reliable results, the frequency of the applied load shall be far enough from the first natural frequency of the free vibration of the beam. The results demonstrate that the integrated displacements in specified nodes through the time of vibration carry enough information about damages in elements and the proposed method can be successfully used for damage detection in beam type structures.

1. Introduction

The main goal of conducting damage detection in structures is to pinpoint the exact location of the damage in structural elements as well as to evaluate damage severity. This will help engineers to assess the remaining service life of the structural members and evaluate the serviceability of the whole structure. The main theory behind most of the damage detection techniques is that any damage will change the mechanical characteristics of a structure which brings about changes in structural responses to the natural excitation.

Vibration-based damage detection techniques have been of interest to researchers during past decades [1]. Based on the natural excitation technique (NExT) [2,3] the cross-correlation function between two responses of a structure subjected to the

* *Corresponding author*. Fax: +1 517 353 7248.
E-mail addresses: mousavimirkalaee.mohsen@gmail.com (M. Mousavi), a.h.gandomi@gmail.com (A.H. Gandomi).

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ambient excitation has the same analytical form as the impulse response (free vibration response of the structure). This rule helps significantly with the identification of the modal parameters of a structure in service. Some researchers have developed some damage detection techniques which consider changes in local or global modal data directly or indirectly [4–6].

Among the modern structural damage detection methods, there are some which are of more interest to researchers such as wavelet analysis, statistically based damage detection techniques, genetic algorithm (GA) and artificial neural network (NN), etc. Although these methods have been studied comprehensively in the literature, there are still several problems which are vital to be dealt with and need to be solved in the modern-type methods, such as: (1) it is still necessary for the structural responses to the environment excitation to be detected; (2) the measured signals are still likely to be contaminated by noise which may mask the information from tiny damage in structures; and (3) the selection and construction of the structural damage index is another issue to be considered.

There are several methods that, using output-only data, are able to identify mechanical system characteristics such as mode shapes, frequencies, etc. The results are extensively used in vibration-based damage detection techniques. However, few studies are focused on direct damage detection from ambient excitation [7]. Most of the damage detection techniques use dynamic characteristics of a structure such as natural frequencies and mode shapes derived from vibration data to conduct damage detection. This will reduce the accuracy of the procedure compared with when the vibration data are directly used.

There is a group of damage detection approaches in the literature lying within the time domain input–output methods. These methods are based on transforming the dynamical problem into an equivalent static one by integrating the input and the output signals in the time domain [8,9]. The output signal is the area under structural reaction graph from the initiation to the end of the vibration due to imposed excitation.

In this study, a novel damage detection method is introduced. The main characteristic of the proposed method is that it just uses translational time history response of structural elements at specified nodes corresponding to the finite element model of the structure. Also, the only required data from the original FE model of the structure is its stiffness matrix. To assess the capability of the proposed method in damage detection a cantilever beam is studied. To that end, a model of a cantilever beam subjected to a sinusoidal load has been modeled in SAP software to derive required data. Then a MATLAB code has been developed to apply introduced algorithm to compute damage indicators. The results demonstrate that the proposed damage detection method can be successfully applied to detect damages in structures.

2. Static equivalent formulation of dynamic vibration of an FE model

The finite element formulation of the dynamic equilibrium of a structure subjected to a dynamic load is written as follows:

$$[\mathbf{M}]\{\ddot{\delta}\} + [\mathbf{C}]\{\dot{\delta}\} + [\mathbf{K}]\{\delta\} = \{f(t)\} \tag{1}$$

where [M], [C] and [K] are mass, damping, and stiffness matrix of the structure, respectively. Also vectors $\{\delta\}$ and $\{f(t)\}$ represent dynamic displacement and force vectors, respectively. Vectors $\{\dot{\delta}\}$ and $\{\ddot{\delta}\}$ are first and second order partial differentiation of the displacement vector toward time. By integrating Eq. (1) in time interval [0,∞] one may obtain:

$$\lim_{t\to\infty}\int_0^t [\mathbf{M}]\{\ddot{\delta}\}\cdot dt + \lim_{t\to\infty}\int_0^t [\mathbf{C}]\{\dot{\delta}\}\cdot dt + \lim_{t\to\infty}\int_0^t [\mathbf{K}]\{\delta\}\cdot dt = \lim_{t\to\infty}\int_0^t [f]\{t\}\cdot dt \tag{2}$$

In Eq. (2) the upper bond of the integration indicates the time of when the structure stops vibrating and evokes a purely physical concept of the infinitive.

Solving Eq. (2) leads to the following equation:

$$\lim_{t\to\infty}[\mathbf{M}]\{\dot{\delta}\}\Big|_0^t + \lim_{t\to\infty}[\mathbf{C}]\{\delta\}\Big|_0^t + [\mathbf{K}]\{\Delta\} = \{\mathbf{F}\} \tag{3}$$

where;

$$\{\Delta\} = \lim_{t\to\infty}\int_0^t \{\delta\}\cdot dt \quad \& \quad \{\mathbf{F}\} = \lim_{t\to\infty}\int_0^t \{f(t)\}\cdot dt \tag{4}$$

Expanding Eq. (3) we have:

$$[\mathbf{M}]\left\{\lim_{t\to\infty}\dot{\delta}(t) - \dot{\delta}(0)\right\} + [\mathbf{C}]\left\{\lim_{t\to\infty}\delta(t) - \delta(0)\right\} + [\mathbf{K}]\{\Delta\} = \{\mathbf{F}\} \tag{5}$$

Considering a zero initial condition for the system leads to this fact that the velocity and displacement of the structure in any position at the beginning of the vibration to be zero. Similarly, at the end of the vibration these values are equal to zero so the terms $\lim_{t\to\infty}\dot{\delta}(t), \dot{\delta}(0), \lim_{t\to\infty}\delta(t)$, and $\delta(0)$ can be neglected. Therefore, Eq. (5) finally takes the following form:

$$[\mathbf{K}]\{\Delta\} = \{\mathbf{F}\} \tag{6}$$

Eq. (6) is known as the static equivalence of Eq. (1). Also terms $\{\Delta\}$ and $\{F\}$ represent the static equivalence of the dynamic force and vibration of the structure.

3. Continuum damage mechanics

Fatigue cracks bring about degradation in material module of elasticity which consequently decreases the stiffness of damaged elements [11]. Accordingly, damage in an element is represented by a parameter α so that $\alpha \in [0,1]$. Consequently, when α is equal to 0 the element is considered intact and when α equals to 1 it is considered that the element is destroyed totally. Therefore, the stiffness matrix of a damaged structure is introduced as follows:

$$[\mathbf{K}_d] = [\mathbf{K}] - [\Delta\mathbf{K}] \tag{7}$$

where;

$$[\Delta\mathbf{K}] = \sum_{i=1}^{n}\alpha_i[\mathbf{K}_i] \tag{8}$$