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Original Research Article

Compressive study of functionally graded plates resting on Winkler–Pasternak foundations under various boundary conditions using hyperbolic shear deformation theory

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ABSTRACT

Equilibrium equations of a functionally graded plate resting on two-parameter elastic foundations are derived using hyperbolic shear deformation theory. This theory takes into account the hyperbolic distribution of transverse shear deformation and satisfies that the corresponding shear stresses equal to zero on upper and lower surfaces of the plate without requiring any shear correction factors. Eight different types of boundary conditions are considered. Governing equations are obtained including the plate-foundation interaction. The present results are compared well with the corresponding available in the literature. Effects of boundary conditions, linear (Winkler) modulus and shear foundation (Pasternak) modulus, gradient index, plate aspect ratio, side-to-thickness ratio on the stresses and deflections are all discussed. It is established that the present model is more accurate than some theories developed previously.

1. Introduction

Functionally graded materials (FGMs) have attracted considerable attention due to high performance, heat resistant capacity and excellent characteristics in comparison with conventional composites. They are microscopically inhomogeneous where the mechanical properties vary continuously and smoothly from one surface to the other, and thus eliminating the stress concentration found in laminated composites. The present FG plates are made from a mixture of metal and ceramic and their materials are nonhomogeneous but often isotropic [1].

Many studies have been performed to analyze the behavior of FGM and sandwich plates. Lanhe [2] has discussed the thermal buckling of a simply-supported moderately thick rectangular FG plate using the first-order shear deformation theory [3]. Zhao et al. [4] have studied the thermal and

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mechanical buckling analysis of FG plates according to the first-order shear deformation theory by using the element-free Ritz method. Yaghoobi and Yaghoobi [5] have studied the buckling analysis of FG sandwich plates resting on two-parameters elastic foundation under various boundary conditions. In their study, an analytical approach (power series Frobenius method) is used to solve the equations of stability, also it is used to reduce the governing equations. Yaghoobi and Fereidoon [6] have discussed thermal and mechanical buckling analysis of FG plates resting on elastic foundations. In their study, a refined nth-order shear deformation theory is proposed [7], formulated and validated for a variety of numerical examples of FG plates resting on two-parameter elastic foundation for the thermal and mechanical buckling responses.

Plates supported by two-parameters elastic foundation commonly encounter technical problems in many structures applications. Various types of elastic foundation models have been proposed to show the interaction between the foundation and the plate structures. The first type is the linear Winkler model (one-parameter) where the foundation consists of a series of separated spring wanting coupling effects between each other [8]. This describe that there is a proportional interaction among the deflection and the external forces of the applied point in the foundation. There are many a applications for Winkler parameter have transact with vibration and buckling of plates [9–11]. The second type is Pasternak (shear) foundation which the separated springs in Winkler foundation are adding by a shear spring to simulate the interactions between them [12]. Pasternak model (i.e. two-parameter elastic foundations) is widely used to show the mechanical behavior of the structure-foundation interactions. Many articles have been discussed the plates resting on one- and two-parameter elastic foundations [13–26].

The varied in boundary conditions and the rises of parameters foundations have significant effects on the behavior of the FG plate. Thus, in this paper the effect of foundations parameters and various types of boundary conditions for the bending response of FG plates under a transverse mechanical load are studied using a hyperbolic shear deformation theory. A general hyperbolic displacement model for deflection and stresses analysis is developed for FG plates resting on two-parameter elastic foundations under eight types of boundary conditions. The shear stresses are lacking on the upper and lower surfaces of the plate. Numerical and graphical results for stresses and deflections are given. An excellent comparison with the available results in the literature is made.

2. FG plate and material properties

Consider a composite plate of uniform thickness h, width b and length a as shown in Fig. 1. The plate is made of an FGM with material properties varying smoothly in the z direction only. The present FG plate is composed from a mixture of ceramic (zirconia) and metal (titanium alloy). At the lower $(z = -\frac{h}{2})$ surface the plate is pure metal, while at the upper $(z = \frac{h}{2})$ surface the plate is pure ceramic. Poisson's ratio is assumed to be constant through the plate thickness and equal to 0.3. We assume that the modulus of elasticity (E) can be determined as [27]:

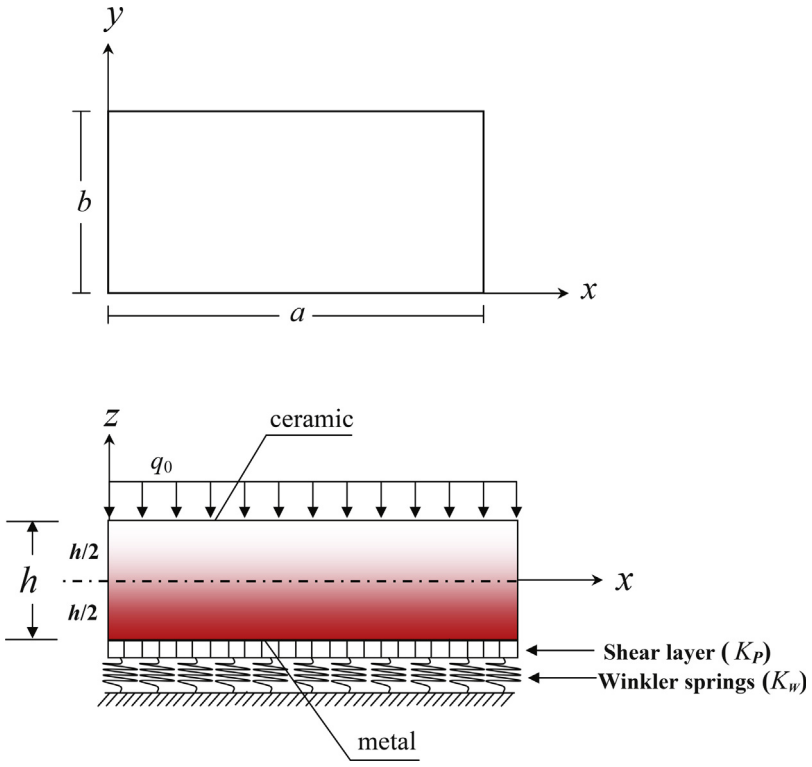

Fig. 1 – Coordinates system of a FG plate resting on Winkler–Pasternak foundations.