ARCHIVES OF CIVIL AND MECHANICAL ENGINEERING 17 (2017) 222–230

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journal homepage: http://www.elsevier.com/locate/acme

Original Research Article

# Tolerance modeling of dynamic behavior of thin plates with dense system of ribs in two directions

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ARTICLE INFO

*Article history:*
Received 18 December 2014
Accepted 24 September 2016
Available online

*Keywords:*
Composite plate
Finite element method
Tolerance averaging technique
Vibrations

ABSTRACT

The subject of this paper are thin composite rectangular plate. The plates are made of two families of ribs and homogeneous material of a matrix. The main feature of the considered plates is, that a distance between the ribs is comparable to the thickness of the plate. The widths of the ribs can vary slowly in the midplane of the plate. This allows you to get a desirable frequency of natural vibrations of the plate. The formulation of averaged model equations is based on the tolerance averaging approach (Wozniak et al. 2008, 2010). The general results of the contribution are illustrated using the analysis of natural vibrations of the plates under consideration. It will be carried out validation of the obtained mathematical model by comparison of results from obtained model equations with results from finite elements method (Abaqus program).



## 1. Introduction

The subject of the contribution is thin composite plate made of homogeneous matrix and dense system of ribs. The aim of this contribution is to propose and apply the averaged mathematical model describing dynamic behavior of these plates. The composite plates are made of two families of thin ribs which axes intersect at right angle and homogeneous matrix which fills the regions between the ribs (Fig. 1a). It is assumed that the width of the beams ($d(x_1, x_2)$, Fig. 2) can vary slowly in the midplane of the plate. Thus, we deal with composite plate having space-varying microstructure. The generalized period $l = \sqrt{l_1 l_2}$ of heterogeneity is assumed to be sufficiently small comparing to the measure of the midplane of the plate. At the same time, it is assumed that the microstructure length parameter $l$ is sufficiently small when compared to the minimum characteristic length dimension of the plate. The fundamental feature of analyzed plates is that the size of the microstructure $l$ is comparable with the thickness of the plate $h$ ($h \cong l$). In the paper Rabenda and Michalak [1] were analysed analogous plates but subjected to normal forces. Plates with the microstructure of this kind are described by differential equations of motion with discontinuous and highly oscillating coefficients. These equations are too complicated to apply to engineering problems. Therefore, a simplified averaged model of these plates will be proposed in which material properties will be represented by functional but smooth effective stiffnesses.

The tolerance averaging approach will be base to formulation of the averaged mathematical model for the analysis of dynamic behavior of these plates. The general mathematical modeling techniques of this approach can be find in books, Woźniak et al. [2,3]. The applications of this technique for the modeling of various dynamic problems of elastic periodic

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http://dx.doi.org/10.1016/j.acme.2016.09.001
1644-9665/

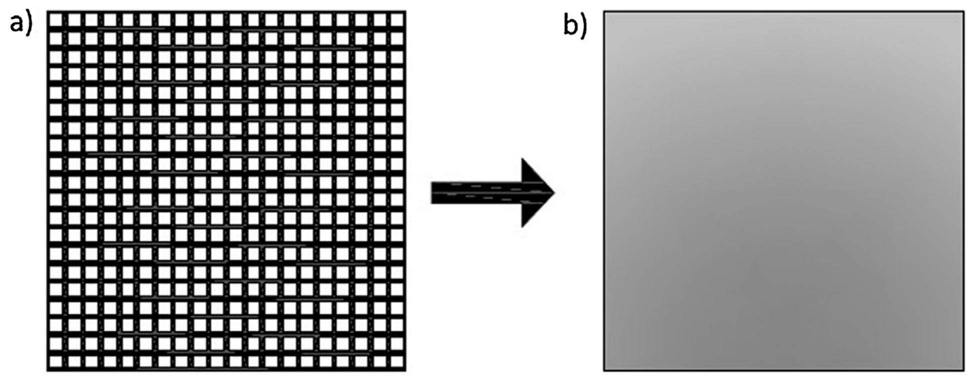


**Fig. 1 – Composite plate (a) at microscopic level and (b) at macroscopic level.**

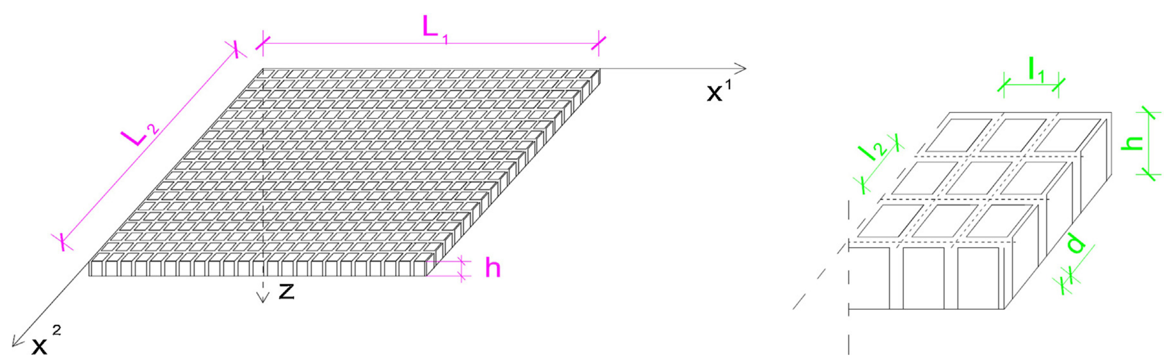


**Fig. 2 – Rectangular plate with dense system of ribs.**

composites are given in a series of papers. Some of the following papers can be mentioned here as examples: in the paper of [4] the one-directional periodic plates in which period length is of the same order as the plate thickness are analyzed. Jeleniewicz and Nagórko analyzed the rectangular composite plate subjected to the plane stress. The elastic plate is reinforced by system of periodically distributed parallel thin ribs. In the paper of [5] higher order vibrations caused by microheterogeneous structure of thin periodic plates were analyzed. In the paper of [6] vibrations of thin plates with initial geometrical imperfections as a model of elastic wavy plates are analyzed. In the paper of [7] the dynamic modeling of visco-elastic composites was discussed. In the paper of [8] the dynamic behavior of honeycomb based composite solids are analyzed.

The averaged models for functionally graded stratified solids, based on the tolerance averaging approach, can be found in the following papers. In the paper of [9] the multilayered composite with functionally graded macroscopic properties was discussed. [10] discussed the dynamic modeling of thin plate made of certain functionally graded materials. The obtained averaged equations of motion were verified by the comparison of the results obtained from the tolerance model and asymptotic model equations with the results from finite element method (Abaqus program). In the paper of [11] was presented 2D tolerance and asymptotic models of dynamic behavior of a plane structure reinforced by a dense system of thin parallel distributed ribs. The thickness of neighboring ribs can smoothly change. [12] studied natural vibrations of thin plate band with non-linear functionally graded material.

In the majority of above mentioned papers the thickness $h$ of the plate is supposed to be much smaller compared to the microstructure length parameter $l = \sqrt{l_1 l_2}$ ($l_1$, $l_2$-dimensions of the cell). Though in the paper of [4] the dimension of cells is of same order as the plate thickness, but the plate is reinforced in only one direction. In opposite to above mentioned papers, in the presented contribution we deal with the plates which are reinforced by two dimensional system of ribs where the microstructure length parameter $l$ is similar compared to thickness $h$ of the plate ($l \cong h$). On a microscopic level we deal with the microheterogeneous plate (Fig. 1a). After averaging on the macroscopic level we deal here with a special case of a functionally graded material (Fig. 1b).

## 2. Direct description

The object of our considerations are rectangular plates shown in Fig. 2. Let us introduce the orthogonal Cartesian coordinate system $Ox_1x_2x_3$ and the time coordinate $t$. Throughout the paper, indices $i, k, l, \ldots$ run over 1, 2, 3 and indices $\alpha, \beta, \gamma, \ldots$ run over 1, 2 and $A, B, C, \ldots$ run over 1, 2. The summation convention holds all aforementioned sub-and superscripts. Setting $\mathbf{x} \equiv (x_1, x_2)$ and $z = x_3$ we assume that the undeformed plate occupies the region $\Omega \equiv \{(\mathbf{x}, z) : -h/2 \le z \le h/2, \ \mathbf{x} \in \Pi\}$, where $\Pi$ is the rectangular plate midplane and $h$ is the plate thickness (Figs. 3 and 4).