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Multiscale simulation of major crack/minor cracks interplay with the corrected XFEM

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ABSTRACT

The present work aims at saving computational cost of multiscale simulation on major crack/minor crack interaction problems. The multiscale extended finite element method (MsXFEM) used for the numerical simulation is developed on multiscale projection technique which enables different scale decomposition, and transition of field variables between different scales. Both macroscale and microscale problems are solved independently and alternatively, in the framework of XFEM. The improvement made in this paper is to employ corrected XFEM on the macroscale level, so that a more accurate boundary condition can be obtained for the microscale problem. The modification leads to a reduced necessary microscale domain size, meanwhile a solution of higher accuracy and enhanced convergence rate can be achieved. The numerical examples of minor cracks near a major one are studied, which show that the effect of minor cracks on major crack can be efficiently captured.

1. Introduction

Recently, industrial engineering structures such as bridges, aerospace structures, engines, buildings, become more and more large-scale and complex, which exhibit multiscale behaviors. Therefore, one needs to investigate the localizing phenomena caused by internal flaws (cracks, holes or inclusions) in order to ensure the reliability of the structures [1,2]. It can be observed that, minor cracks in the vicinity of a major crack tip have significant influence on the major crack propagation, as they can result in either crack shielding or crack amplification, which should be taken into account to predict the fatigue life of the components.

For the numerical simulation of the localizing phenomena in single-scale analysis, an extreme locally refined mesh is

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required, which inevitably lead to high computational cost and poorly conditioned equations. Therefore, multiscale methods were introduced since it can dramatically reduce the computational cost, which offer great promise in modeling localizing phenomenon such as local nonlinearity or stress concentration caused by microscopic flaws in complex structures. After finite element method (FEM) been widely applied to numerical simulation of large scale structures, the origin idea of multiscale method was to enhance the performance of FEM over the entire region or some subdomain. Hirai et al. [3] proposed a family of so-called zooming methods, using refined finite element meshes for the local regions containing stress concentrations. In a superposition multiscale approach [4], the global and local parts were modeled independently by different meshes, and then superimposed to provide a final solution which satisfies the compatibility equations. However, this method might present computational weakness such as robustness or low convergence rate. An alternative approach is the multiple scale expansion technique based on homogenization of field variables at each scale [5,6]. This method is very useful to provide a global solution for the bulk of the structure, yet fail to produce a local solution accurately for the localizing phenomena. Other concepts of multiscale methods include variational multiscale method [7], domain decomposition method [8,9], and concurrent methods [10,11] etc. Among these methods, the multiscale projection method introduced by Loehnert et al. [12,13] offers great advantages over others since it produces accurate solution for both macroscale and microscale problems and it is ease to implement in large programming platform. In this method, transition of field variables (displacement, stress strain) between different scales can be achieved, which make the multiscale projection method ideal for the macro/micro cracks interaction simulation.

When the evolving cracks are numerically simulated, it is preferred that, the mesh is independent of the physic geometry and the initial mesh remains unchanged when the crack propagates. Various methods have been developed to overcome the difficulty caused by remeshing, such as the boundary element method [14,15], the mesh free method [16,17], the numerical manifold method [18] and the extended finite element method (XFEM) [19–21]. Among them, XFEM is highly competitive and widely used, since it enable the mesh totally independent of the physic discontinuities with minimal added degrees of freedom. In XFEM, the discontinuities and stress singularity are modeled by locally enriching the classical FEM approximation with additional functions.

The XFEM have already been successfully applied to multiple minor discontinuities at a single scale, for example, large array of cracks by Budyn et al. [22], microcrack/macrocrack simulation by Guidault et al. [23], and multiple minor flaws in functionally graded materials by Singh et al. [24]. Lately, a number of researchers have conjugated the multiscale method with XFEM for minor flaws simulation. For example, based on LATIN method, Guidault et al. [25] proposed a multiscale XFEM for crack propagation, in which only in vicinity of the discontinuity the mesh is refined. Bosco et al. [26] developed a fully coupled micro-macro solution strategy where the solution procedure on the macroscopic level is based on XFEM. Loehnert et al. [13] conducted major crack and minor cracks interplay simulation by multiscale projection in conjugation with standard XFEM, the role of microfield in the macrofield can be emphasized, the involved parameters influencing the results were investigated. It is concluded in his paper that, it is important to determine an adequate large domain size of the microstructure. In standard XFEM, the blending elements which blend the locally enriched part and the other major part compromise the overall convergence and local accuracy. Special treatments [27,28] have been proposed to overcome such arisen problem. The most effective among them is corrected XFEM proposed by Fries [29], in which a ramp function is introduced into the enrichment functions of the blending elements, the modification achieves optimal convergence and high accuracy. Therefore, the purpose of the present paper is to modify the multiscale projection method by using corrected XFEM on the macroscopic level, in order to reduce the necessary domain size of microstructure, and enhance the convergence rate.

This paper is organized as follows. In Section 2, the framework of corrected XFEM is described, in which, the modification of standard XFEM is detailed. In Section 3, the traditional multiscale projection method is presented, which involves the transition of field variables at different scales and separation of the structural details. In Section 4, first a beam containing a single major crack is simulated to demonstrate the efficiency and accuracy of the modified method, then a family of cases as major crack in presence of minor cracks are investigated and compared with previous researches. Section 5 gives the conclusions and an outlook to future work.

2. Formulation of standard and corrected XFEM

2.1. *Displacement approximation of standard XFEM*

In XFEM, the major continuum part is modeled by FE approximation, while the local discontinuity is modeled by additional enrichment functions which are constructed by the means of the partition unity theory. For a 2-D body containing cracks, the displacement approximation of standard XFEM [30] can be written as:

$$\hat{u}(x) = \sum_{j \in A} N_j(x) u_j + \sum_{k \in M} N_k(x)[H(x) - H(x_k)] a_k + \sum_{l \in I} N_l(x) \sum_{\alpha=1}^{4} [\beta_\alpha(x) - \beta_\alpha(x_l)] b_l^\alpha \tag{1}$$

As marked in Fig. 1, A is the set of all nodes in the mesh; M is the set of nodes belonging to those split elements which intersects with the crack; I is the set of nodes belonging to the tip element which contains the crack tip. If a node belongs both to split element, and tip element, then, the node belongs to I set.

In Eq. (1), u_j is the classical finite element displacement; $N_j(x)$, $N_k(x)$ and $N_l(x)$ are standard FE shape functions, which technically do not need to be identical. $H(x)$ is the Heaviside function used to model the discontinuity in displacement, which takes +1 on one side of the crack surface and −1 on the other side. a_k are the nodal unknowns enriched on the M set of nodes. $\beta_\alpha(x)$ (α = 1–4) are four asymptotic crack tip branch