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Inverse estimation of spatially and temporally varying heating boundary conditions of a two-dimensional object

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In many dynamic heat transfer situations, the temperature at the heated boundary is not directly measurable and can be obtained by solving an inverse heat conduction problem (IHCP) based on measured temperature or/and heat flux at the accessible boundary. In this study, IHCP in a two-dimensional rectangular object is solved by using the conjugate gradient method (CGM) with temperature and heat flux measured at the boundary opposite to the heated boundary. The inverse problem is formulated in such a way that the heat flux at heated boundary is chosen as the unknown function to be recovered, and the temperature at the heated boundary is computed as a byproduct of the IHCP solution. The measurement data, i.e., the temperature and heat flux at the opposite boundary, are obtained by numerically solving a direct problem where the heated boundary of the object is subjected to spatially and temporally varying heat flux. The robustness of the formulated IHCP algorithm is tested for different profiles of heat fluxes along with different random errors of the measured heat flux at the opposite boundary. The effects of the uncertainties of the thermophysical properties and back-surface temperature measurement on inverse solutions are also examined.

1. Introduction

In some dynamic heat transfer situations, the surface heat flux and temperature histories of a solid cannot be measured directly with thermal sensors. For example, during surface heat treatment processes, the treated surface may be unsuitable for attaching a sensor in order to avoid intervention of the thermal manufacturing process. Similar situations can also be found during the reentry of space vehicles where the heated surface temperature is so high that a thermal sensor can be destroyed by ablation. Under such circumstances, the heated (front) surface temperature can be determined indirectly by solving an inverse heat conduction problem (IHCP) [1,2] based on the transient temperature and/or heat flux measured on the opposite (back) surface.

Solutions of IHCPs are very challenging because they are mathematically classified as *ill-posed*. Although some analytical solutions are available for the solution of IHCPs (e.g., [3,4]), the majority of the solution method relies on the numerical approach, in which the inverse problem is re-stated as a least-squares minimization problem over the whole-time domain or in sequential time intervals. A regularization parameter is introduced to stabilize the inverse solutions [5]. However, the optimal value of the regularization parameter is often difficult to acquire. Alifanov's iterative regularization technique [6] is an alternative approach for traditional regularization scheme. In this technique, the regularization procedure is performed during the iterative processes and thus the determination of the optimal regularization parameter is not required.

The conjugate gradient method (CGM) belongs to the category of iterative regularization techniques. It can improve the convergence rate of inverse estimation by choosing the direction of descent as the linear combination of the gradient direction at current iteration with the direction of descent at previous iteration [7]. Due to its excellent self-adjusting, global convergence property, the CGM has been extensively used to solve multidimensional and non-linear IHCPs (e.g., [8–12]). However, most of the CGM algorithms in the past were based on temperature measurement data (e.g., [13–18]) since the temperature can be measured with less uncertainty compared to the heat flux [19–22]. Little work has been done for the inverse numerical algorithm using heat flux measurement data. Furthermore, in heat treatment and aerospace applications, the surface heat flux may be delivered in a periodic way in time and with a non-uniform profile in space due to the spinning or moving of the target. This may pose extra challenges on the inverse solutions, so it is necessary to determine which CGM formulation is more appropriate for such applications.

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Nomenclature

C	volume specific heat, J/(m^3 K)
$d^k(y,t)$	direction of descent at iteration k, which is sometimes expressed in vector form $\mathbf{d^k}$
f	frequency of the heat flux at heated boundary, Hz
h	convection heat transfer coefficient, W/(m^2 K)
i_m	total number of heat flux sensors
k	thermal conductivity, W/(m K)
L_x, L_y	object lengths in x and y directions, respectively, m
q	heat flux, W/m^2
q_{sur}	heat flux at heated boundary, W/m^2
$q_1(y,t)$	observed heat flux at heated boundary which is sometimes expressed in vector form $\mathbf{q_1}$, W/m^2
$q[L_x,y,t;\ \mathbf{q}_1]$	computed heat flux at opposite boundary, W/m^2
$\Delta q[L_x,y,t;\ \mathbf{d}^k]$	heat flux variation, which is sometimes simplified as $\Delta q(\mathbf{d^k})$, when the boundary heat flux is subjected to a perturbation $\Delta q_1(y,t) = d^k(y,t)$, W/m^2
S	objective function
$\nabla S[\mathrm{q}_1^k]$	gradient direction of objective functional at iteration k
t	time, s
Δt	time step, s
T	temperature, K
T_b	boundary temperature, K
T_∞	ambient temperature, K
$T_1(y,t)$	front surface temperature, K
$\Delta T[L_x,y,t;\mathbf{d^k}]$	temperature variation, which is sometimes simplified as ΔT, when the boundary heat flux is perturbation is $\Delta q_1(y,t) = d^k(y,t)$, K
w	1/e radius of Gaussian-profile boundary heat flux, m
x, y	spatial coordinate variables, m
$Y(y,t)$	measurement data (temperature or heat flux) with errors at opposite boundary obtained by numerical simulations
$Y_{qLexact}(y,t)$	measurement heat flux without errors at opposite boundary obtained by numerical simulations, W/m^2
$Y_{qL}(y,t)$	measurement heat flux at opposite boundary, W/m^2
$Y_{TL}(y,t)$	measurement temperature at opposite boundary, K

Greek symbols

α	surface absorptivity
β^k	search step size at iteration level k
χ	tolerance used to stop the CGM iteration procedure
δ	Dirac delta function
ε	surface emissivity
ϕ	standard deviation of heat flux or temperature measurements
γ^k	conjugate coefficient at iteration level k
$\lambda(x,y,t)$	Lagrange multiplier
σ	Stefan–Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W/(m^2 K^4)
ω	a random variable having a normal distribution with zero mean and unitary standard deviation

Superscripts

k	iteration level

Subscripts

0	initial
f	final

Recently, the authors proposed a robust and error-insensitive 1-D IHCP formulation to reconstruct the front-surface heating condition with back-surface measurement data [23]. It has been shown that the most accurate solution can be obtained by choosing the front-surface heat flux as the unknown function and using the temperature measurement data as the boundary condition at back surface while the heat flux measurement data are employed in the objective function. However, the work presented in [23] is only for 1-D geometry. The objective of this paper is to extend the previous 1-D algorithm to a 2-D formulation and check its performance in estimating the surface heating transients caused by spatially and temporally varying boundary conditions.

2. Model description

To illustrate the methodology of the inverse heat transfer algorithm employed in this study, a two-dimensional rectangular object, as shown in Fig. 1, is considered. Initially, the object is uniformly at temperature T_0 and is subjected to a high-intensity heat flux q_{sur} with any arbitrary profiles (Fig. 1 uses Gaussian profile as an example, and w is 1/e radius) from $t = 0^+$ at its front boundary ($x = 0$). The purpose of this study is to demonstrate the effectiveness and accuracy of the proposed IHCP formulation in reconstructing the observed heat flux $q_1(y,t)$ and temperature $T_1(y,t)$ at the front boundary of this 2-D target with temperature-dependent thermophysical properties, based on the measured temperature and heat flux at the back boundary ($x = L_x$). Due to the fact that temperature measurement contains much less errors compared to the heat flux measurement [19–22], the back-boundary temperature $Y_{TL}(y,t)$ is used as the boundary condition and the back-boundary heat flux $Y_{qL}(y,t)$ is employed in the objective function.

2.1. Direct problem

The direct problem can be expressed as follows:

$$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k(T)\frac{\partial T}{\partial y}\right] \tag{1}$$

$$T = T_0 \quad \text{for} \quad 0 \le x \le L_x,\ 0 \le y \le L_y,\ t = 0 \tag{2}$$

$$-k(T)\frac{\partial T}{\partial x} = q_1(y,t) \quad \text{for} \quad x = 0,\ 0 \le y \le L_y,\ t > 0 \tag{3}$$

$$T = Y_{TL}(y,t) \quad \text{for} \quad x = L_x,\ 0 \le y \le L_y,\ t > 0 \tag{4}$$

$$-k(T)\frac{\partial T}{\partial y} = 0 \quad (\text{adiabatic}) \text{ or } T = T_b \quad (\text{first kind B.C.})$$
$$\text{for} \quad y = 0 \quad \text{and} \quad L_y, 0 \le x \le L_x, t > 0 \tag{5}$$

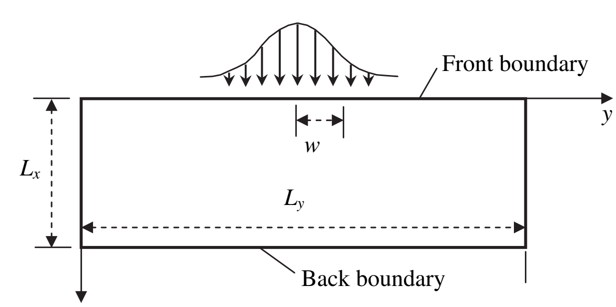

Fig. 1. Physical model.