International Journal of Thermal Sciences 49 (2010) 1763–1775

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Laminar fully developed flow through square and equilateral triangular ducts with rounded corners subjected to H1 and H2 boundary conditions

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ARTICLE INFO

Article history:
Received 25 January 2008
Received in revised form
22 March 2010
Accepted 23 March 2010
Available online 26 May 2010

Keywords:
Laminar
Fully developed
Square
Triangular
Rounded corner

ABSTRACT

The present paper deals with the evaluation of pressure drop and heat transfer characteristics of laminar fully-developed flow through ducts of square and equilateral triangular cross sections with rounded corners, for both H1 and H2 boundary conditions. The dimensionless radius of curvature (R_c) of both type of ducts is varied from zero to the maximum possible value (1 for square duct and $1/\sqrt{3}$ for triangular duct). The solutions for velocity and temperature are considered in the form of a harmonic series. The constants of the series are evaluated by 'least square technique'. From the velocity and temperature solutions, *fRe* and *Nu* are calculated. The results show that for square duct, at lower values of R_c, both *fRe* and *Nu* increase rapidly with R_c and for higher values of R_c, both *fRe* and *Nu* asymptotically assume their values corresponding to that for the circular duct. For triangular ducts, *Nu* shows a similar behaviour. *fRe*, on the other hand, shows a similar behaviour only for lower values of R_c. At moderate R_c, *fRe* attains its maximum value around $R_c \approx 0.35$ and with further increase in R_c, *fRe* drops slightly and finally tends to its value corresponding that of a circular ducts in an asymptotic manner. For both type of ducts, *Nu* for H1 boundary condition is always higher than that for H2 boundary condition. It is also observed that the straight portion of the duct is always more effective than the circular duct, where as, the effectiveness of the rounded portion, which is always less than the circular duct, increases with the increase in R_c. Correlations, in two different forms, are obtained for all the cases and they show excellent agreement with the computed data.

1. Introduction

Study of thermal hydraulic characteristics for laminar fully-developed flow through ducts of different cross section assumes importance in design of heat exchangers of existing geometric configuration, as well as, for development of newer types of flow passages in them in order to achieve better thermal hydraulic performance. The primary objective of the heat exchanger designer is to work with duct geometries that yield (i) a high value of heat transfer area to volume ratio, (ii) a high value of heat transfer coefficient and (iii) a corresponding low value of friction factor. As a result, substantial research work is focused towards development of compact and efficient duct geometries, those satisfy the above criteria.

In compact heat exchangers, due to smaller system dimensions, the hydraulic diameter is low, which, for most of the practical cases, yields essentially laminar flow. It is therefore essential to investigate the thermal hydraulic behaviour of ducted flows, particularly in the laminar regime. These studies find its relevance in different areas, such as, aerospace, nuclear, chemical and process industries, biomedical, electronics and instrumentation.

In most of the heat exchangers in service, especially in shell and tube type, generally circular duct is used. However, it may be noted here that a polygonal duct (number of sides = n), offers higher surface area to volume ratio as compared to a circular duct. This ratio increases with the decrease in n. As a result, triangular ducts offer largest surface area to volume ratio and for the circular duct, for which n is infinite, it is the least. Hence, from the view point of compactness, the triangular duct should be the most preferred geometry and the square ducts should be the next choice. Circular ducts, on the other hand, should have the least priority.

Offering maximum compactness, i.e., highest surface area to volume ratio, however, is not the sole criterion for selection of duct geometry. A designer should also look into the overall thermal hydraulic behaviour of the flow through the ducts. In this regard, it may be mentioned here that the sharp corners of the polygonal ducts offer least effective heat transfer surface as the bulk flow tends to by-pass them. The corners, in fact, act as hot spots, where

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doi:10.1016/j.ijthermalsci.2010.03.012

Nomenclature

A	Half side of the duct, m
A_c	Cross-sectional area, m^2
a_j, b_j	j-th. constants for velocity solution
C	Constant in momentum equation, (ms)$^{-1}$
C_p	Specific heat at constant pressure, J/kg K
c_j, d_j	j-th. constants for temperature solution
d_h, D_h	Hydraulic diameter ($4A_c/P$), m
f	Friction factor
h	Heat transfer coefficient, W/m^2 K
k	Thermal conductivity, W/m K
m	Number of boundary points
Nu	Nusselt number
n	Number of terms in the series
p	Pressure, N/m^2
P_h, P_w	Heated and wetted perimeter, m
$\dot{q}''$, $\dot{q}''_p$	Average and local heat flux for H2 boundary condition, W/m^2
$\dot{Q}'$	Heat input per unit axial length, W/m
Re	Reynolds number
r_c, R_c	Radius of curvature, m
r	Radial coordinate, m
s	Distance along the duct wall, m
t	Temperature, °C
t_b, t_w	Bulk and wall temperatures, °C
T	Transformed temperature
w	Axial velocity, m/s
w_m	Average axial velocity, m/s
w^*	Transformed axial velocity, m^2
z	Axial coordinate, m
α	Thermal diffusivity, m^2/s
Δp	Pressure drop along axial length L, N/m^2
μ	Dynamic viscosity, N-s/m^2
ν	Kinematic viscosity, m^2/s
ρ	Density, kg/m^3
θ	Angular coordinate, rad
Γ	The boundary of the duct

the temperature is significantly higher than the rest of the duct surface. As a result the overall heat transfer performance of the sharp cornered ducts reduces to a certain extent. Since the sharp corners of the polygonal ducts exhibit ineffective heat transfer characteristics and as a result affect the overall thermal hydraulic performance, such ducts with rounded corners may offer a possible remedy. The present study is devoted to address this issue.

Extensive research work has been carried out by numerous researchers to study the thermal hydraulic behaviour of laminar flow through ducts of various geometries as the problem is of practical importance and relevant to heat exchanger industries. Upto 1978, exhaustive literature review has been presented by Shah and London [1]. Subsequently, Hartnett and Kostic [2] have compiled the existing experimental and numerical data for Newtonian and non-Newtonian fluid flow through rectangular ducts in both laminar and turbulent regimes. As more and more articles are still being published in this area, it is apparent that further monographs are essential on this topic. A detailed review of all the literature covering various thermal hydraulic aspects of flow through ducts of different geometries, subjected to various possible boundary conditions, is left out of the scope of the present work. However, in the present context, literatures are reviewed only on works covering generalized techniques for flow and heat transfer through arbitrary shaped ducts.

As reported by Shah and London [1], there are various methods available for solution of relevant momentum and energy equations applicable for laminar fully-developed ducted flows, ranging from analytical treatments to computational solutions by finite difference/element techniques. Some of these methods are applicable only for simple geometries. For example, 'exact solution' is possible, only when the concerned geometry and the boundary conditions are relatively straightforward. On the other hand, some of the methods, like numerical solutions, are applicable to almost all kind of duct geometries and boundary conditions, although they are quite costly. As far as the analytical or semi-analytical methods are concerned, in the past, various researchers have used the 'Conformal Mapping' technique [3], the 'Generalised Integral Transform' technique [4], the 'Variational Method' [5] and the 'Series Solution Method' [6–13]. Among these various approximate semi-analytical methods, solutions by 'Point-Matching-Methods' and 'Least Square Methods', those fall into the general class of series solution method, require special mention, as these methods are very general in nature and are applicable to fully-developed flow through ducts of very complex shape, as long as a constant wall heat flux type of boundary condition is used.

In general, for fully-developed ducted flow, if the dependent variables are suitably transformed, a Laplace and a Poisson equation can represent the momentum and the energy equations respectively. The energy equation, after decomposing the dependent variable (temperature) into complementary and particular solutions, can be further transformed to a Laplace equation. The general solution of Laplace equation is obtained by a linear combination of harmonic functions in the form of a truncated infinite series, consisting of N (starting from zero) terms. Therefore, the method involves solution of $n = 2N + 1$ unknown coefficients. The solutions exactly satisfy the governing equations, although the implementation of boundary conditions requires special treatment.

In order to solve the coefficients, when n number points are chosen on the boundary, and same number of unknown coefficients is solved, the method is termed as 'point-matching method'. The final solution (the series) satisfies the boundary conditions at the chosen points, although at other points on the boundary, errors are expected. Sparrow et al. [6,7] have applied the point matching method, using algebraic–trigonometric polynomials, for longitudinal flow over array of cylinders. Cheng and Jamil [8] have adopted the point matching technique to study flow and heat transfer in cylindrical ducts with diametrically opposite flat sides.

In the 'least square method' more than n points (say, m, usually $2n$ to $3n$ points) along the boundary are employed to determine n unknown coefficients in the truncated infinite series. The coefficients of the series are evaluated by solving m linear algebraic equations by minimising the squared error of the boundary conditions at m ($m > n$) chosen points. Thus, in the least square method, the exact fit at the discrete boundary points (as in 'point-matching method') is sacrificed to obtain a better fit to the boundary as a whole. The least square method has been employed Ratkowsky and Epstein [9] and Hagen and Ratkowsky [10] to study laminar flow in regular polygonal ducts with circular centered cores and in cylindrical ducts with regular polygonal cores respectively. The least square approximation was first adopted by Sparrow and Haji-Sheikh [11] employing Gram-Schmidt orthonormalisation to study flow and heat transfer in arbitrary shaped ducts. However, they furnished results only for circular ducts and sectors. Subsequently, Shah [12] obtained various averaged parameters like friction factor in the form of fRe and Nusselt number for flow through ducts of various shapes, for example, rectangular, isosceles triangular,