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MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition

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ABSTRACT

A numerical approach has been used to study the heat and mass transfer from a vertical plate embedded in a porous medium experiencing a first-order chemical reaction and exposed to a transverse magnetic field. Instead of the commonly used conditions of constant surface temperature or constant heat flux, a convective boundary condition is employed which makes this study unique and the results more realistic and practically useful. The momentum, energy, and concentration equations derived as coupled second-order, ordinary differential equations are solved numerically using a highly accurate and thoroughly tested finite difference algorithm. The effects of Biot number, thermal Grashof number, mass transfer Grashof number, permeability parameter, Hartmann number, Eckert number, Sherwood number and Schmidt number on the velocity, temperature, and concentration profiles are illustrated graphically. A table containing the numerical data for the plate surface temperature, the wall shear stress, and the local Nusselt and Sherwood numbers is also provided. The discussion focuses on the physical interpretation of the results as well their comparison with the results of previous studies.

1. Introduction

The study of hydromagnetic boundary layer flow with heat and mass transfer over a vertical surface embedded in a porous medium is important in many engineering situations such as the concurrent buoyant upward gas-liquid flow in packed bed electrodes [1], sodium oxide-silicon dioxide glass melt flows [2], reactive polymer flows in heterogeneous porous media [3], electrochemical generation of elemental bromine in porous electrode systems [4] and the manufacture of intumescent paints for fire safety applications [5]. A comprehensive survey of magneto-hydrodynamic studies and their technological applications can be found in the book by Moreau [6]. Several interesting computational studies of reactive MHD boundary layer flows with heat and mass transfer have appeared in the recent years [7–12]. Chamkha and Khaled [13] reported similarity solutions for hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media. Merkin and Chaudhary [14] used an asymptotic analysis to study natural convection boundary-layer flow on a vertical surface with exothermic catalytic chemical reaction and concluded that the flow is controlled by the activation energy, the heat of reaction and Prandtl and Schmidt numbers. Makinde [15] used a shooting numerical technique to analyze MHD boundary-layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux at the plate surface. Except for a few, boundary layer flows have been studied using either a constant surface temperature or a constant heat flux boundary condition. Aziz [16] has recently studied the Blassius flow over a flat plate with a convective thermal boundary condition and established the condition which the convection heat transfer coefficient must meet for a similarity type solution to exist. In a contemporaneous study, Bataller [17] studied the boundary layer flow over a convectively heated flat plate with a radiation term in the energy equation. Because the convective boundary condition is more general and realistic especially with respect to several engineering and industrial processes like transpiration cooling process, material drying, etc., it seems appropriate to use the convective boundary condition to study other boundary layer flow situations. This paper considers MHD mixed convection from a vertical plate with heat and mass transfer and a convective boundary condition at the plate. It is assumed that the plate is embedded in a uniform porous medium and is exposed to a transverse magnetic field. The problem is solved numerically and results are presented for the velocity, temperature, and concentration profiles together with the local skin friction, the plate surface temperature and the local heat and mass transfer rates. The results are believed to be applicable to realistic engineering situations cited earlier.

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Nomenclature

$(\overline{u},\overline{v})$	velocity-components
g	gravitational acceleration
K	permeability parameter
C_p	specific heat at constant pressure
$\overline{T}$	fluid temperature
$\overline{T}_\infty$	free stream temperature
$\overline{T}_f$	hot fluid temperature
Gc	mass transfer Grashof number
M	magnetic field parameter
Pr	Prandtl number
Sh	Sherwood number
Bi	Biot number
k	thermal conductivity
(u, v)	dimensionless velocity-components
D	mass diffusivity
v_0	wall suction velocity
$\overline{C}_\infty$	free stream concentration
$\overline{C}_w$	concentration at plate surface
B_0	magnetic field strength
Gr	thermal Grashof number
U	free stream velocity
Ec	Eckert number
Sc	Schmidt number
v_0	constant wall suction
$(\overline{x},\overline{y})$	coordinates

Greek symbols

θ	dimensionless temperature
ϕ	cimensionless concentration
β	thermal expansion coefficient
σ	electrical conductivity
$\overline{\beta}$	concentration expansion coefficient
ρ	fluid density
υ	fluid kinematic viscosity
λ	reaction rate parameter
γ	reaction rate coefficient

2. Mathematical model

We consider a steady, laminar, hydromagnetic coupled heat and mass transfer by mixed convection flow of a cold fluid at temperature T_∞ over an infinite vertical plate embedded in a porous medium. It is assumed that the left surface of the plate is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f. The cold fluid on the right side of the plate is assumed to be Newtonian and electrically conducting. Except for the density, all other fluid properties are assumed to be independent of temperature and chemical species concentration. A uniform magnetic field of strength B_0 is imposed normal to the plate (along the y-axis) as shown in Fig. 1. Since the magnetic Reynolds number is very small for most fluid used in industrial applications, we assume that the induced magnetic field is negligible.

Under the Boussinesq and boundary-layer approximations, the momentum, energy balance and concentration equations can be written in as [10,15–17],

$$-v_0\frac{\partial\overline{u}}{\partial\overline{y}} = \upsilon\frac{\partial^2\overline{u}}{\partial\overline{y}^2} + g\beta(\overline{T}-\overline{T}_\infty) + g\overline{\beta}(\overline{C}-\overline{C}_\infty) + \frac{\upsilon}{\overline{K}}(\overline{U}-\overline{u}) + \frac{\sigma B_0^2}{\rho}(\overline{U}-\overline{u}), \tag{1}$$

$$-v_0\frac{\partial\overline{T}}{\partial\overline{y}} = \frac{k}{\rho C_p}\frac{\partial^2\overline{T}}{\partial\overline{y}^2} + \frac{\upsilon}{C_p}\left(\frac{\partial\overline{u}}{\partial\overline{y}}\right)^2 + \frac{\sigma B_0^2}{\rho C_p}(\overline{U}-\overline{u})^2, \tag{2}$$

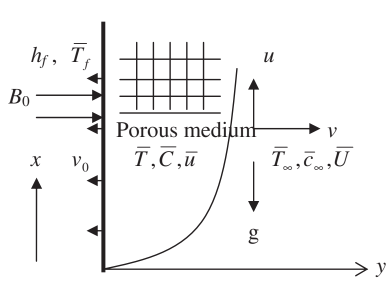

Fig. 1. Flow configuration and coordinate system.

$$-v_0\frac{\partial\overline{C}}{\partial\overline{y}} = D\frac{\partial^2\overline{C}}{\partial\overline{y}^2} - \gamma(\overline{C}-\overline{C}_\infty), \tag{3}$$

where $\overline{x}$ and $\overline{y}$ are, respectively, the directions along and perpendicular to the surface, $\overline{u}$ is the velocity component along the plate, $\overline{U}$ is the uniform free-stream velocity, v_0 is the constant suction velocity, g is the acceleration due to gravity, ρ, υ, k and σ are, the fluid density, kinematic viscosity, the thermal conductivity and the electrical conductivity, respectively. The local temperature is $\overline{T}$, $\overline{C}$ is the species concentration and the subscript ∞ denotes free-stream conditions. The other parameters are the coefficient of volume expansion for heat transfer β, the coefficient of volumetric expansion with respect to species concentration $\overline{\beta}$, the molecular diffusivity D, and the reaction rate coefficient γ. The appropriate boundary conditions for this flow are

$$\overline{u} = 0, \quad -k\frac{\partial\overline{T}}{\partial\overline{y}} = h_f\left(\overline{T}_f-\overline{T}\right), \quad \overline{C} = \overline{C}_w, \quad \text{at} \quad \overline{y} = 0, \tag{4}$$

$$\overline{u}\to\overline{U}, \quad \overline{T}\to\overline{T}_\infty, \quad \overline{C}\to\overline{C}_\infty, \text{ as } \overline{y}\to\infty, \tag{5}$$

where the convective heating process at the plate is characterized by the hot fluid temperature $\overline{T}_f$ and the heat transfer coefficient h_f.

Introducing the following dimensionless quantities,

$$y=\frac{v_0\overline{y}}{\upsilon}, u=\frac{\overline{u}}{v_0}, U=\frac{\overline{U}}{v_0}, \theta=\frac{\overline{T}-\overline{T}_\infty}{\overline{T}_f-\overline{T}_\infty}, \phi=\frac{\overline{C}-\overline{C}_\infty}{\overline{C}_w-\overline{C}_\infty}, Sc=\frac{\upsilon}{D},$$
$$Gr=\frac{g\beta(\overline{T}_f-\overline{T}_\infty)\upsilon}{v_0^3}, Pr=\frac{\rho\upsilon C_p}{k}, Gc=\frac{g\overline{\beta}(\overline{C}_w-\overline{C}_\infty)\upsilon}{v_0^3}, \lambda=\frac{\gamma\upsilon^2}{Dv_0^2}, \tag{6}$$
$$M=\frac{\sigma B_0^2\upsilon}{\rho v_0^2}, Bi=\frac{h_f\upsilon}{kv_0}, K=\frac{v_0^2\overline{K}}{\upsilon^2}, Ec=\frac{v_0^2}{C_P\left(\overline{T}_f-\overline{T}_\infty\right)}.$$

Eqs. (1)–(5) may be written in dimensionless form as follows.

$$-\frac{du}{dy} = \frac{d^2u}{dy^2} + Gr\theta + Gc\phi + M(U-u) + \frac{U-u}{K}, \tag{7}$$

$$-\frac{d\theta}{dy} = \frac{1}{Pr}\frac{d^2\theta}{dy^2} + Ec\left(\frac{du}{dy}\right)^2 + MEc(U-u)^2, \tag{8}$$