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Heat transfer to power-law dilatant fluids in a channel with a built-in square cylinder

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ABSTRACT

In this study, heat transfer to power-law dilatant fluids from a long square cylinder (heated) confined in a channel in the steady flow regime is investigated. The effects of Reynolds number, Prandtl number and flow behavior index on the heat transfer characteristics of a cylinder is examined for the range of conditions $1 \leqslant Re \leqslant 45$, $1 \leqslant n \leqslant 2.0$ and $1 \leqslant Pr \leqslant 100$ (the maximum Peclet number being 4000) for a fixed blockage ratio, $\beta = 1/8$. The variation of the local Nusselt number on the individual surfaces of the square obstacle for the constant wall temperature (CWT) and uniform heat flux (UHF) boundary conditions prescribed on the surface of the square obstacle are presented. Likewise, the representative isotherm plots for the two classical thermal boundary conditions are shown. The average Nusselt number and the heat transfer factor (j_h) have also been calculated. Irrespective of the value of the flow behavior index, the value of the local Nusselt number at each corner of the square cylinder increases with an increase in the Reynolds and/or Prandtl number. The average Nusselt number increases monotonically with an increase in the Reynolds and/or the Prandtl number. Finally, simple heat transfer correlations have been provided for the range of conditions covered here.

1. Introduction

The fluid flow across cylinders (square or circular cross-section) exhibits a rich variety of flow regimes depending upon the value of the Reynolds number, power-law index and the flow domain (confined or unconfined) [1–15]. Most multiphase mixtures, i.e., foams, suspensions, emulsions, etc. and high molecular weight polymeric systems, i.e., solutions, melts, blends, etc. exhibit shear-thinning (pseudo-plastic fluids, $n < 1$) and/or shear-thickening (dilatant fluids, $n > 1$) behaviors under appropriate flow conditions and the simple non-Newtonian power-law model is able to describe satisfactorily both the shear-thinning and shear-thickening behaviors over moderate ranges of shear rates [16–18]. In spite of such a wide occurrence of power-law fluids, very little is known about the heat transfer characteristics of a heated cylinder of square or circular cross-section confined in a channel.

Indeed, there are only a few numerical heat transfer studies from the circular cylinder to power-law fluids in both confined and unconfined flow configurations [7,14,15]. Soares et al. [7] investigated the forced convection heat transfer from an unconfined circular cylinder for the range $5 \leqslant Re \leqslant 40$, $1 \leqslant Pr \leqslant 100$ and $0.5 \leqslant n \leqslant 1.4$. They used the stream-function/vorticity formulation to solve the governing momentum and thermal energy equations. They reported that the effect of power-law index on the local Nusselt number was less pronounced for the constant temperature condition than that for the uniform heat flux condition. The average Nusselt number was found to be a decreasing function of power-law index; however, for lower value of the Peclet number such dependence was found to be less pronounced. Along the same lines, Bharti et al. [14] investigated the heat transfer from an unbounded circular cylinder to power-law fluids for $0.6 \leqslant n \leqslant 2.0$, thereby covering the weak shear-thinning and strong shear-thickening fluids. The numerical investigations have been carried out by using finite volume method (FVM) for $1 \leqslant Pr \leqslant 1000$, but for the same range of Reynolds number as used by Soares et al. [7]. The pseudo-plastic fluids show higher heat transfer than that for Newtonian ($n = 1$) and dilatant fluids. Simple heat transfer correlations have also been established based on the numerical results. As far as channel confined circular object case is concerned, there is only one heat transfer study due to Bharti et al. [15] in the steady cross-flow regime. They investigated the effect of two blockage ratios (1/4 and 5/8) on the heat transfer characteristics of a circular cylinder for the range $1 \leqslant Re \leqslant 40$, $1 \leqslant Pr \leqslant 100$ and $0.2 \leqslant n \leqslant 1.8$ by using the commercial CFD package Fluent. They found the enhancement in the rate of heat transfer with the increasing degree of shear-thinning behavior; however, an opposite trend was observed in dilatant fluids for a fixed value of the blockage ratio. Aside from these studies, there have been numerous analyses based on the usual boundary layer approximation [9] and most

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Nomenclature

b	side of a square cylinder .... m
c_p	specific heat of the fluid .... $\mathrm{J\,kg^{-1}\,K^{-1}}$
h	local heat transfer coefficient .... $\mathrm{W\,m^{-2}\,K^{-1}}$
$\bar{h}$	average heat transfer coefficient .... $\mathrm{W\,m^{-2}\,K^{-1}}$
j_h	Colburn factor for heat transfer $(= Nu/(Re\,Pr^{1/3}))$
k	thermal conductivity of the fluid .... $\mathrm{W\,m^{-1}\,K^{-1}}$
L_1	length of the computational domain .... m
L_2	height of the computational domain .... m
m	power-law consistency index .... $\mathrm{Pa\,s^n}$
N_i	number of grid points in the x-direction
N_j	number of grid points in the y-direction
n	power-law index
n_s	normal direction to the surface of the cylinder
Nu	average Nusselt number $(= \bar{h}b/k)$
Nu_L	local Nusselt number $(= hb/k)$
p	pressure $(= p'/(\rho U_{\max}^2))$
Pe	Peclet number $(= Re\,Pr)$
Pr	Prandtl number $(= (mc_p/k)(U_{\max}/b)^{n-1})$
q_w	constant heat flux on the surface of the cylinder .... $\mathrm{W\,m^{-2}}$
Re	Reynolds number $(= \rho U_{\max}^{2-n} b^n/m)$
T	temperature $(= (T' - T_\infty)/(T'_w - T_\infty)$ or $(T' - T_\infty)/(q_w b/k))$
T_∞	temperature of the fluid at the inlet .... K
T'_w	constant wall temperature at the surface of the cylinder .... K
u	component of the velocity in the x-direction $(= u'/U_{\max})$
$U_{\max}$	maximum velocity of the fluid at the inlet .... $\mathrm{m\,s^{-1}}$
v	component of the velocity in the y-direction $(= v'/U_{\max})$
x	stream-wise coordinate $(= x'/b)$
X_d	downstream face distance of the cylinder from the outlet
X_u	upstream face distance of the cylinder from the inlet
y	transverse coordinate $(= y'/b)$

Greek symbols

β	blockage ratio $(= b/L_2)$
δ	size of the control volume clustered around the cylinder .... m
Δ	size of the control volume far away from the cylinder .... m
ε	component of the rate of deformation tensor $(= \varepsilon'/(U_{\max}/b))$
η	power-law viscosity $(= \eta'/\eta_0)$
η_0	reference viscosity $(= m(U_{\max}/b)^{n-1})$ .... Pa s
I_2	second invariant of the rate of deformation tensor $(= I'_2/(U_{\max}/b)^2)$
ρ	density of the fluid .... $\mathrm{kg\,m^{-3}}$
τ	extra stress component $(= \tau'/(\eta_0 U_{\max}/b))$

Subscripts

f, t, r	front, top and rear faces of the square cylinder, respectively
i	index used in the x-direction
j	index used in the y-direction
N	Newtonian
w	surface of the cylinder
∞	inlet condition

Superscript

$'$	dimensional variable

of these along with the scant experimental results have been reviewed in [16].

Similarly, there are only three previous numerical studies dealing with the heat transfer from a square obstacle to power-law fluids in the steady cross-flow regime [2,3,8]. Gupta et al. [2] examined the heat transfer from the confined cylinder to power-law liquids for $5 \leqslant Re \leqslant 40$, $0.5 \leqslant n \leqslant 1.4$ and $1 \leqslant Pr \leqslant 10$ ($5 \leqslant Pe \leqslant 400$) for both CWT and UHF conditions for a fixed blockage ratio of 1/8. They implemented the finite difference method (FDM) on a uniform staggered grid arrangement without any clustering around the cylinder and near the channel walls. Subsequently, Paliwal et al. [3] studied the heat transfer characteristics of an unconfined square obstacle over the identical ranges of the Reynolds number, Prandtl number and the flow behavior index as covered by Gupta et al. [2]. In gross, shear-thinning fluid behavior seems to facilitate heat transfer whereas shear-thickening behavior impedes it irrespective of whether the cylinder is confined or unconfined. In yet another similar study, Nitin and Chhabra [8] extended the work of Gupta et al. [2] for the heat transfer to power-law liquids from a cylinder of rectangular cross-section for the same range of conditions. Interestingly, it is worthwhile to point out here that in all these studies [2,3,8], a relatively coarse and uniform mesh was used having only about 10 cells on the each side of the cylinder. Therefore, these preliminary results are probably not very accurate, as confirmed by subsequent more accurate and detailed studies [12,19–24]. Recently, the effects of power-law index on the flow characteristics across the square cylinder have been studied for three values of the blockage ratio (1/8, 1/6 and 1/4) in [19]. In another recent study, Dhiman et al. [25] have investigated the mixed convection to power-law fluids from an isolated square cylinder in an unconfined flow configuration. Mixed convection effects have also been investigated in the channel confined steady flow regime in [26]. Therefore, it can be summarized here that very limited numerical results are available in the literature on the cross-flow of non-Newtonian dilatant fluids past a confined square cylinder. Thus, the objective of this work is to study the effects of Reynolds number and Prandtl number on the heat transfer across a confined long cylinder with square cross-section for dilatant fluids in the steady regime. Further, owing to high viscosity levels of many substances of multi phase nature and/or of high molecular weight (e.g., pulp and paper suspensions, polymer melts and biological process engineering applications, etc.), this study includes the results for power-law index up to 2.

2. Problem formulation

Two-dimensional, steady and incompressible fluid flow across a square cylinder confined in a channel is studied here, as shown in Fig. 1. The cylinder of side b is exposed to a parabolic velocity profile with maximum velocity, $U_{\max}$ and uniform temperature, T_∞ at the inlet. The non-dimensional distance from the inlet plane to the front surface of the cylinder is X_u/b, and the distance between the rear surface of the cylinder and the exit plane is X_d/b. The total non-dimensional length of the computational domain is L_1/b in the axial direction. The non-dimensional height of the computational domain is L_2/b in the lateral direction.

The continuity, x- and y-components (assuming negligible buoyancy effects) of Cauchy's equations [19] and of the energy