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An integral approach for simulation of vapour film dynamics around a spherical surface

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ABSTRACT

The dynamics of a thermally driven vapour film around a solid sphere has been investigated here with both the sphere and the annular film surrounded by a large water pool. Integral models based on constant and variable vapour-phase densities have been developed here for studying a spherico-symmetric phase change problem for two immiscible phases, vapour and liquid around a hot sphere. Governing equations for both liquid and vapour phases are converted into a set of non-linear ODEs. Effects of distinct density on interface condition and density variation of vapour phase are taken into account both in energy equation of vapour phase and also in interfacial mass and energy balance. The present models have been validated with available analytical, incompressible Volume of Fluid (VOF) and experimental results of growth and collapse of either bubble or vapour film. A simple model, based on scale analysis, was evolved that successfully captured the non-monotonic growth of the film, as observed by the more detailed models under certain degree of liquid subcooling. In addition, the case of very small thermal boundary layer in the liquid side has been successfully studied for which the VOF model required very fine grid. It has been observed that the effect of density variation in the integral model results in marginally higher film growth at higher temperature. However, the effect of radiation on the film growth was found to be quite substantial. The integral model not only incorporates the effects of vapour-phase temperature variation and radiation exchange of heat but also is computationally several-fold efficient with respect to the VOF model.

1. Introduction

Two immiscible phases separated by an interface are present in many engineering problems involving phase change. If the phase interface between the liquid and its vapour is associated with sharp temperature gradient, evaporation and transport of mass take place across the interface causing it to move. Plesset and Zwick [1] investigated the inertia-controlled spherico-symmetric dynamics of a vapour bubble in a pool of water. Their prediction is reasonable during the initial period, if the liquid superheat is high or the system pressure is low. A perturbation analysis of the energy equation in presence of a thermal boundary layer in the liquid side revealed that towards the terminal phase the dynamics is thermally maintained. Mikic et al. [2] obtained a unified expression of the radius of a saturated vapour bubble combining the results of simple scale analyses for both the inertial and thermally controlled regimes. Their prediction exhibited good agreement with those of Dalle Donne and Ferranti [3], except for very small liquid superheats.

Prosperetti and Plesset [4] extended the solution of Mikic et al. [2] by introducing scaling variables to describe growth over a wide range of superheats. Lee and Merte [5] and Robinson and Judd [6] carried out detailed numerical analysis of spherico-symmetric bubble growth in superheated liquid. Naude and Mendez [7] numerically analysed the collapse of vapour bubble in subcooled liquid. Riznic et al. [8] assumed spherico-symmetry in order to carry out an integral analysis associated with a vapour bubble for its collapse in a subcooled liquid pool and growth in a superheated liquid pool. While Prosperetti and Plesset [4] and Robinson and Judd [6] included the inertial effect on thermal growth in their scale analysis, Naude and Mendez [7] and Riznic et al. [8] neglected the effect of liquid momentum to lay emphasis only on thermally driven growth and collapse. Avdeev and Zudin [9] extended the regime of applicability of the solution by Mikic et al. [2] by incorporating the effects of heat inflow and inertial reaction of the liquid to bubble expansion in the energy transport and Rayleigh equations for the bubble. Their approximate analytical solution captured the features of certain experimental findings for the first time. The solution exhibited smooth transition from one parametric regime to another along with asymptotic approaches to the limiting solutions.

Closed form solutions for the thermally induced dynamics have been obtained by neglecting the density difference between the

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Nomenclature

a_2	time varying constant	T	time ..... s
C_p	specific heat of fluid ..... J/kg K	u	transformed variable $T.r$ ..... K m
J	evaporative mass flux across interface ..... kg/m^2 s	*Greek letters*	
Ja	Jakob number, $Ja = \rho_l C_p \lvert(T_s - T_\infty)\rvert/\rho_{v_s} L$, dimensionless	v	velocity ..... m/s
L	latent heat of phase change ..... J/kg	ρ	density ..... kg/m^3
R	interface radius ..... m	λ	thermal conductivity ..... W/m K
R_0	initial interface radius ..... m	α	thermal diffusivity ..... m^2/s
R_g	gas constant ..... J/kg K	δ	liquid-side thermal boundary layer thickness ..... m
R_m	sphere/melt radius ..... m	η	$(r - R_m)/(R - R_m)$, dimensionless
$\dot{R}$	interface velocity ..... m/s	ε	emissivity, dimensionless
Ste	Stefan number, $Ste = C_p(T_m - T_s)/L$, dimensionless	γ	surface tension co-efficient ..... N/m
T	temperature ..... K	σ	Stefan–Boltzmann constant ..... W/m^2 K^4
T_s	saturation temperature ..... K	*Subscripts*	
T_∞	far liquid temperature ..... K	l	liquid phase
h	enthalpy ..... J/kg	v	vapour phase
p	pressure ..... N/m^2	m	melt (sphere)
q_{rad}	radiation heat flux ..... W/m^2	s	saturation condition
R	radial co-ordinate ..... m		

phases for situations where a particular phase remains at saturation condition corresponding to the system pressure. These approximations lead to analytical solution in standard simple geometries that are known as solution to Stefan problem [10]. Application of Stefan problem is mainly pertinent to phase change in material storage device, where the interface movement arises from either melting or solidification of a particular phase. Caldwell and Kwan [11] solved the outward solidification problem for different geometries by perturbation methods.

If the phases are not saturated and, in addition, the difference in densities between two phases is considered, the problem differs from the Stefan-like situation. The interface boundary condition calls for the solution of energy equation in both phases with the bulk-phase motion activated by the motion of interface. In Cartesian geometry Carslaw and Jaeger [12] analysed the effect of difference in densities, with one phase at saturation temperature. For the validation of their code, both Welch and Wilson [13] and Esmaeeli and Tryggvasson [14] used the similarity solution for the outward movement of a planar liquid–vapour interface. In these formulations, the effect of distinct densities is crucial and convection of energy in the superheated bulk liquid is explicitly accounted for. However, the vapour phase was considered as saturated.

Another situation demands attention when the vapour film is between a molten metal in the interior and liquid at the exterior. Though such dynamic studies are of interest for the prediction of rapid collapse of a vapour film during molten fuel-coolant-interaction [15,16] and a number of other technical applications [15], these remain limited till date. Bejan et al. [17] and Dan et al. [18] investigated the growth of a vapour film around a molten metal drop immersed in a saturated liquid. They considered radiative heat transfer between the metal drop and the water, considering the vapour layer to be non-participating. In addition, they considered the convective heat transfer due to the radial velocity field induced by temporal and spatial variation of vapour phase density. However, this convective effect was not considered in their estimation of evaporation rate through mass and energy balance at the interface. Ghosh et al. [19] adopted a generalised incompressible Volume of Fluid (VOF) based methodology for prediction of collapse and growth of a thin vapour film layer formed around a hot sphere immersed in a large liquid pool.

In the present work, an ODE-based spherico-symmetric approximate integral model has been developed that can predict the collapse and growth of a thin layer of a vapour film formed around a hot sphere immersed in a large liquid pool. The VOF-based methodology [19], developed earlier for a similar configuration is computationally intensive and is difficult to extend to multiple drops in a real system. Our present model can capture the case of distinct but constant density for each phase. An interesting prediction of initial decrease of film size followed by growth from the VOF-based investigation has been used to validate the present formulation, in absence of any experimental result. A scale analysis has been performed to predict the duration of the initial decay and compared with the prediction of the integral analysis. For a rigorous validation of the integral formulation, a few other problems have also been considered for which either of analytical, experimental and other numerical results are available.

In contrast to the VOF study [19], the integral study brings out the additional effects of the density variation and consequent velocity field in the vapour phase. Also following Bejan et al. [17], the contribution of radiation heat flux from the hot sphere to steam–water interface has been incorporated, assuming the vapour film to be non-participating. In addition, the effect of radial velocity field induced by density variation in the vapour phase has been included in the equation for mass and energy balance at the interface that yields the expression for evaporation rate. The developed model has been validated against some of the available analytical and numerical solution for moving boundaries with one of the phases at saturated condition. Studies have been carried out next considering none of the phases as saturated. A detailed parametric study of the dynamics of film around the hot sphere in a subcooled liquid has finally been investigated.

2. Mathematical model

In this section the system of equations have been derived using mass and energy conservation principles, the liquid–vapour interface condition and boundary conditions for the bulk liquid phase of infinite expanse and the vapour phase of the limited expanse. In the ensuing derivation, the liquid phase has been considered incompressible along with spherico-symmetric assumption for all primary variables. Two alternative formulations have been developed, one assuming constant density for the vapour phase and the