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Constructal design of non-uniform X-shaped conductive pathways for cooling

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ABSTRACT

This paper applies constructal design to discover the configuration that facilitates the access of the heat that flows through non-uniform X-shaped pathways of high-conductivity material embedded within a square-shaped heat-generating medium of low-conductivity to cooling this finite-size volume. The objective is to minimize the maximal excess of temperature of the whole system, i.e. the hot spots, independent of where they are located. The total volume and the volume of the material of high thermal conductivity are fixed, but the lengths of the pathways and the angles between the pathways can vary. The configuration was optimized for four degrees of freedom: the two angles between the pathways, and the two ratios between the lengths. It was found numerically that the performance of the non-uniform X-shaped pathways is approximately 10% better than the performance of the uniform X-shaped pathways, i.e. X-shaped with equal lengths and thicknesses. When compared to the configuration calculated for a simpler I-shaped blade (i.e. a single pathway of high thermal conductivity material beginning in the isothermal wall and ending of such a way that there is spacing between the tip of the pathway and the insulated wall) the non-uniform X-shaped pathways configuration performs 56% better.

1. Introduction

Constructal theory is the view that the generation of flow configurations is a physics phenomenon that can be based on a physics principle (the constructal law). The constructal law states that for a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that it provides easier access to the currents that flow through it [1–4].

Constructal design is the method based on constructal law to discover the configurations that facilitates the access of the flow currents. The method applies the objective and constraints principle, i.e. the geometry can change freely as we vary the degrees of freedom subjected to the constraints. The applicability of this method to engineered flow systems has been widely discussed in recent literature, e.g. in designing cavities and assembly of fins [5–9].

The problem of cooling with minimal thermal resistance or minimal excess of temperature a finite volume that generates heat at every point has been largely studied in the literature [10–32]. Particularly, conductive heat transfer is a very effective way to cooling electronic devices. Bejan and Lorente [2] have shown that when length scales drop a certain level the heat transfer by conduction is the winner to cooling solid bodies when compared to convective heat transfer. The reason is that the available space should be occupied by material that contributes to the purpose of the cooling system [9] and not by ducts that channel the fluid. Important contribution to constructal theory literature, constructal heat-generating body problems, and cooling of electronics can also be found in the references, [33–52].

Constructal theory has also been applied successfully to the study of heat trees at micro nano scales and nanofluids [53–56]. It is also worth to mention the work presented in Refs. [57–64] which took entransy dissipation rate minimization or maximum thermal resistance minimization as the objective function to discover the best shape for different geometries.

This paper relies on constructal design to study numerically a square elemental volume which generates heat uniformly per unit of volume. The body is cooled by a heat sink at temperature T_0 that is located in the rim. The objective is to minimize the maximal excess of temperature $T_{max} - T_0$. To facilitate the access of the elemental volume heat currents, it is inserted non-uniform

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Nomenclature

A	area [m^2]
D	thickness [m]
k	thermal conductivity [W m^{-1} K^{-1}]
L	length [m]
q	heat current [W]
q'''	heat uniformly at volumetric rate [W m^{-3}]
T	temperature [K]
V	volume [m^3]
x, y	coordinates [m]
W	width [m]

Greek symbols

α	angle between the inferior branch of the X and the x axis
β	angle between the superior branch of the X and the x axis
θ	dimensionless temperature, Eq. (7)
ϕ	area fraction

Subscripts

max	maximal
min	once minimized
mm	twice minimized
mmm	three times minimized
mmmm	four times minimized
opt	optimal
oo	twice optimized
ooo	three times optimized
oooo	four times optimized
p	path (blades) of high thermal conductivity
0	isothermal wall, single blade
1	superior X-shaped branches
2	inferior X-shaped branches

Superscript

$\tilde{()}$	Dimensionless variables, Eqs. (5)–(9), (11)–(13) and (15)

X-shaped pathways of higher thermal conductivity instead of a single path [10–12]. The radial design (named X-shaped geometry [13]) was chosen based on the successful application of this configuration in heating water and fuel distribution in the landscape. Details about this example of organized flow system can be found in Refs. [2,65,66]. It is made the assumption of two-dimensional problem for the sake of simplicity. The ratio between the thermal conductivity of the elemental volume and the thermal conductivity of the X-pathways, as well as the area fraction are two design parameters of the conductive composite.

2. Mathematical model

Consider the conducting body shown in Fig. 1. The configuration is two-dimensional, with the third dimension (W) sufficiently long in comparison with the length L of the total volume. There are X-shaped pathways of high thermal conductivity k_p material

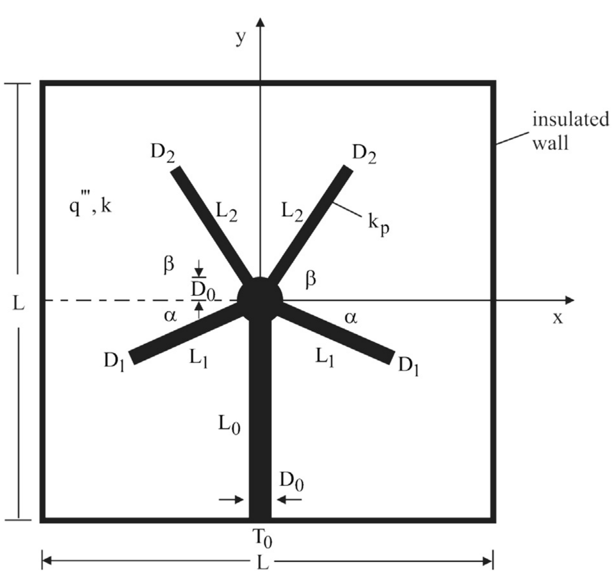

Fig. 1. Body of low conductivity and heat generation with non-uniform X-shaped pathways of higher conductivity.

embedded in the body of lower thermal conductivity k. The solid body generates heat uniformly at the volumetric rate q''' (W/m^3). The outer surfaces of the solid are perfectly insulated. The generated heat current ($q'''AW$) is removed by the heat sink located in the rim of the body at temperature T_0.

The objective of the analysis is to determine the optimal geometry (L_1/L_0, L_2/L_0, α, and β) that is calculated by minimizing the dimensionless maximal excess of temperature $(T_{max} - T_0)/(q'''A/k)$.

According to constructal design, this optimization can be subjected to two constraints. The first constraint is the total area,

$$A = L^2 \tag{1}$$

and the second constraint is the approximate area occupied by the high conductivity material

$$A_p = \pi D_0^2 + D_0 L_0 + 2D_1 L_1 + 2D_2 L_2 \tag{2}$$

Eqs. (1) and (2) can be expressed as the area fraction

$$\phi = \frac{A_p}{A} \tag{3}$$

Due to pure observation it was noted that another geometric constraint emerged and it was given by

$$\frac{L}{2} = L_0 + D_0 \cos(\pi/6) \tag{4}$$

The analysis that delivers the maximal excess of temperature as a function of the geometry consists of solving numerically the steady heat conduction with heat generation equation along the lower conductivity k-region,

$$\frac{\partial^2 \theta}{\partial \tilde{x}^2} + \frac{\partial^2 \theta}{\partial \tilde{y}^2} + 1 = 0 \tag{5}$$

and the steady heat conduction without heat generation equation in the k_p-region