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One dimensional transient heat conduction in segmented fin-like geometries with distinct discrete peripheral convection

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ABSTRACT

This study presents the solution of transient heat conduction in a composite extended surface whose periphery is exposed to convection and whose thermophysical properties experience discontinuities along the longitudinal direction. The solution development uses the natural analytic approach and formats the description so that the constants of integration of each of the composite segments are expressed in terms of the previous segment's eigenfunctions. This allows the solution to be "built" in a very systematic and sequential manner. A three-segment case study is also provided to show the simplicity and applicability of the implementation of this solution.

1. Introduction

This study is concerned with the analytic solution to the transient, one-dimensional conduction of heat in the composite domain. The nature of the composite domain is such that it is composed of numerous layers and that each layer may be characterized by its own individual thermophysical parameter values. When, between any adjacent composite layers, these parameter values vary, a discontinuity is introduced at the interface. With each additional layer, the solution becomes increasingly algebraically complicated. This is because the solution of *each* individual composite layer must carry information of *all* the discontinuities acting at *all* of the composite interfaces. This information is passed between the layers through the interface conditions and is stored in the constants of integration of each layer. Some analytic studies of pure conduction in the composite slab have made efforts to consider the representation of these constants in such a way that greatly reduces the solution's algebraic complexity.

The treatment of transient pure conduction in composite media has been addressed in the past to find exact solutions in the Cartesian, the spherical, and the cylindrical coordinate systems. Exact solution procedures follow either the orthogonal expansion technique or the Laplace transform method. The Laplace transform method is especially suitable to problems in which the thickness of at least one of the composite layers may be described as semi-infinite or for capturing the solution at very early times [1–4]. For geometrically finite composite systems, orthogonal expansion techniques have been used to develop multi dimensional steady solutions in the comprehensive works [5,6], and for transient problems, the transient solutions of one dimensional domains [1,7–12] and of transient multi-dimensional domains [13,14], in conjunction with inverse methods [15,16], and with transient heating and transient boundary conditions [16,17]. Further applications of the analytic solution in the composite domain include variations of transient conduction that address transport in biological media and diffusion-advection in porous media [18–26]. The solutions to transient diffusion and transient conduction in the composite slab of an arbitrary number of "*N*" layers have been presented in previous studies [11,17]. However these solutions are often presented in such a way that the algebraic expressions involved are very complex. This increases the potential for making typographical errors when the symbolic forms of these solutions are transcribed. There have been successful attempts to develop the solution of the composite domain such that the eigenfunctions (and the associated constants of integration) of each layer are explicitly described in terms of the neighboring layers [9,27,28]. This approach lends itself to a solution that can be presented in an algebraically simple, methodical, and programmable manner because it circumvents the need for the implicit simultaneous solutions of one boundary condition and all of the interface conditions.

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Nomenclature

a, b	Constants of integration
c	Specific heat capacity
h	Convection heat transfer coefficient
k	Thermal conductivity
m	Ratio of resistance of heat transfer by conduction along the axis to that of peripheral convection $m_i^2 = (L^2hP/kA_c)_i$
t	Dimensional time variable
t_O	Time constant
x	Dimensionless spatial coordinate $x_i = x_i^+/L_i$
x^+	Dimensional spatial coordinate
A_c	Cross sectional area
E	Error
F	Dimensionless initial condition
K	Inverse conduction resistance $K_i = (k/L)_i$
H_O	Biot number at tip of first segment (1) $H_O = h_O L_1/k_1$
H_L	Biot number at tip of last segment (N) $H_L = h_L L_N/k_N$
j	Imaginary unit: $j = (-1)^{1/2}$
L	Segment length
L_C	Cross sectional characteristic length
P	Perimeter length
T	Temperature
T_∞	Local ambient temperature at segment periphery
T_O	Local ambient temperature of segment 1 exposed end
T_L	Local ambient temperature of segment N exposed end
ΔT_i	Normalized difference of ambient temperatures between adjacent segments $i = 2 \ldots N$: $\Delta T_i = (T_{\infty i} - T_{\infty i-1})/(T_O - T_L)$
ΔT_L	Normalized difference of ambient temperatures at last segment (N) $\Delta T_L = (T_L - T_{\infty N})/(T_O - T_L)$
ΔT_O	Normalized difference of ambient temperatures at first segment (1) $\Delta T_O = (T_O - T_{\infty 1})/(T_O - T_L)$
X_i	Eigenfunction of segment "i" (used in the evaluation of the eigenvalues)
$X_{i,n}$	Eigenfunction of segment "i" associated with the "nth" eigenvalues (used in the evaluation of the series solution)
Greek letters	
Γ	Solution time dependent component
Φ	Dimensionless separation variable $\Phi_i(x_i,\tau) = X_i(x_i)\cdot\Gamma_i(\tau)$
Ψ	Steady Solution Component
α	Thermal diffusivity $\alpha_i = (k/\rho c)_i$
δ	Fourier number $\delta_i = t_O\alpha_i/L_i^2$
μ	Segment eigenvalue
ρ	Density
τ	Dimensionless time $\tau = t/tO$
ϕ	Dimensionless segment temperature $\phi_i = (T_i - T_{\infty i})/(T_O - T_L)$
Repeated subscripts	
i	Segment index integer
n, m	Eigenvalue index integer
F	Associated with the initial condition
N	Number of segments or referring to last segment
R	The reference segment (the segment with a minimum or zero value of the product of δm^2)

This paper will use a similar approach and extend the previous work done in composite systems to a more complex application: transient heat conduction along the composite extended surface. The extended surface introduces additional terms and additional potential for discontinuities that must be addressed in order to extend the previous work. The discussion presented here begins with a description of the extended surface in the composite domain. As the solution is developed, this paper will show significant deviations from the solutions of pure conduction that include the handling of the governing equations and the evaluation of the eigenvalues. These deviations are discussed in detail and the final solution is formulated in an algebraically simple manner.

1.1. Segmented fin

One dimensional transient heat conduction in extended surfaces and fin like structures is typically characterized by some oblong geometry whose periphery is exposed to convection. A simple depiction of this is provided in Fig. 1 where the heat transfer at the base is enhanced by an increase in the area of the wetted surface. The mathematical description of heat conduction in the fin can be represented by the differential equation [29]:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^{+2}} - \frac{hP}{kA_c}(T - T_\infty) \quad (1)$$

where A_C is the fin cross sectional area and P is the fin perimeter length.

The use of Eq. (1) is restricted to situations in which the cross sectional temperature gradient at any longitudinal position, x^+, is negligible. This is relevant when the local cross sectional Biot number, $\mathrm{Bi}_C = hL_C/k$, is small: that is, when the resistance to conduction along the cross section is much smaller than resistance to the peripheral convection heat transfer. A negligible cross sectional temperature gradient also implies that the conduction resistance is much lower along the cross section than it is along the axial direction, and this can be anticipated when the ratio of the characteristic lengths, L_C/L, is small.

Variations of heat conduction in fins and similar structures are found in many engineering applications including: electronics cooling, heat exchangers, and thermal regulation of turbine blades. A great deal of analytic and numerical work addresses the topic which is presented in its steady form in introductory texts on heat transfer, and in its transient form in advanced texts introductory and advanced texts on heat transfer [30,31].

Conventional derivations and solutions of Eq. (1) are limited to several requisite assumptions that are detailed in ([30], pp 86). The most limiting of these assumptions are that:

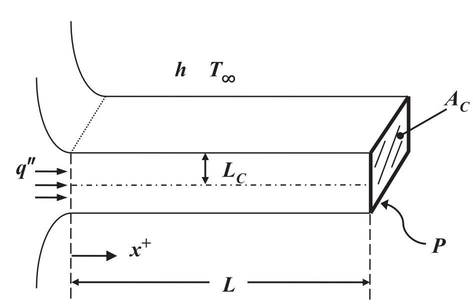

Fig. 1. Extended surface exposed to convective heat transfer at its periphery.