Building and Environment 134 (2018) 181–190

Contents lists available at ScienceDirect

Building and Environment

journal homepage: www.elsevier.com/locate/buildenv

# Calibration of simplified building energy models for parameter estimation and forecasting: Stochastic versus deterministic modelling

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ARTICLE INFO

*Keywords:*
Calibration
Uncertainty
MCMC
Kalman filter

ABSTRACT

Due to the ill-posedness of many inverse problems, parameter estimates are often prone to a possibly large uncertainty, caused by a series of errors and approximations in the experimental and modelling work. Stochastic state-space models for time series modelling incorporate a term of process noise that represents system error; most studies on building thermal model calibration however employ deterministic models that overlook this error.

This paper investigates how accounting for modelling errors affects the results of model calibration. Several simplified models are defined to simulate the indoor temperature of an experimental test cell. Some models include process noise and others do not. The parameters of each model are then learned repeatedly by using several training datasets from the test cell. The MCMC algorithm is used for training. The robustness of parameter estimates between independent trainings is evaluated. Then, the forecasting ability of the deterministic and stochastic options are compared, in terms of accuracy and robustness. Results show that stochastic modelling considerably increases the uncertainty of parameter estimates, but ensures their consistency between separate trainings, whereas deterministic models are less robust and offer a less reliable forecasting.

## 1. Introduction

The calibration of simplified building thermal models using in-situ measurements is now a widespread research topic [1]. It is a type of inverse problems, as the user attempts to identify the causes of a physical phenomenon by observing its consequences: typically, observing the evolution of indoor temperature leads to the estimation of external sollicitations or envelope properties. It is solved as an optimisation problem, where the objective is the minimisation of the deviation between measurements and predictions from a model [2,3]. Such calibration is commonly performed for two general types of applications: the characterisation of the intrinsic building performance [4–9] or other physical values; the identification of a model for predictive purposes [10–12], for instance in the aim of model predictive control [13–16]. In the first case, the model should be based on some physical representation of reality in order to assess physical parameter values. In the second, a black-box model is suitable as it is used primarily for predictive purposes.

Due to the ill-posedness of many inverse problems [17], parameter estimates are prone to a possibly large uncertainty, caused by a series of errors and approximations in the experimental and modelling work [18,19]. First, the model is an approximation of the real system: this model discrepancy may result from missing physics, overlooked input variables, numerical approximations, erroneous hypotheses, etc. Secondly, experimental uncertainty arises from inaccurate or intrusive sensors. Accounting for these errors when solving an inverse problem allows guaranteeing the value of estimated parameters within certain bounds [1].

Most of the time, the inverse problem of parameter characterisation is formulated supposing an unbiased model [18]. According to this hypothesis, there exists a set of parameter values that will allow the model to accurately simulate reality, and the only deviation between its output and experimental observations is measurement noise. This hypothesis is exceedingly optimistic, especially when models used for the characterisation of building thermal behaviour are simplified resistor-capacitor (RC) structures [3]. Accounting for modelling approximations is essential for the legitimacy of calibrated models and the interpretability of their parameters. One possible way to do so is using stochastic differential equations, solved with a Kalman filter for the estimation of states [20]. Another option for the quantification of model uncertainty is to calibrate a discrepancy function in an iterative model updating procedure [21,22].

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https://doi.org/10.1016/j.buildenv.2018.02.043
Received 19 December 2017; Received in revised form 8 February 2018; Accepted 26 February 2018
Available online 01 March 2018

The literature offers many applications of parameter estimation and forecasting with stochastic models [23–27], but no direct comparison with their deterministic counterpart. According to [26]: *stochastic models give more reproducible results and less bias, because random effects due to process and measurement noise are not absorbed into the parameter estimates but specifically accounted for by the noise terms.* Separately, [28] stated and demonstrated that *an analysis that does not account for model discrepancy may lead to biased and over-confident parameter estimates and predictions*. The target of the present paper is to show this effect, in the case of a simple building and lumped capacitance models, made of a network of resistors and capacitors (denoted RC models). The process noise is included in the formulation of some of these models (denoted stochastic models), and excluded in others (deterministic models). The parameters of each model are then learned repeatedly by using several training datasets from an experimental test cell. The robustness of parameter estimates between independent trainings is evaluated. Then, the forecasting ability of the deterministic and stochastic options are compared, in terms of accuracy and robustness.

Sec. 2 briefly recalls the theory of filtering, forecasting and inference in state space models, applied to building modelling. Sec. 3 extends the questioning of the paper shown above, and presents the experimental and numerical methodology to answer it. Sec. 4 then shows the results of this study.

## 2. Inference in state space models for building modelling

### 2.1. Linear state space models

The presented study considers the particular (yet quite widespread and flexible) case of linear, Gaussian, time-invariant, discrete-time state space models [20]:

$$\mathbf{x}_t = \mathbf{F}_\theta\,\mathbf{x}_{t-1} + \mathbf{G}_\theta\,\mathbf{u}_t + \mathbf{w}_t \tag{1}$$

$$\mathbf{y}_t = \mathbf{H}_\theta\,\mathbf{x}_t + \mathbf{v}_t \tag{2}$$

where $t$ is a discrete time coordinate. The terms of this system are denoted as such:

- $\mathbf{x}_t$ is a $p$-dimensional vector of state variables;
- $\mathbf{y}_t$ is a $q$-dimensional vector of observations, or output variables;
- $\mathbf{u}_t$ is a $r$-dimensional vector of inputs variables;
- $\mathbf{w}_t \sim \mathcal{N}(0, \mathbf{Q})$ is the process noise;
- $\mathbf{v}_t \sim \mathcal{N}(0, \mathbf{R})$ is the observation noise.

In addition to the covariances $\mathbf{Q}$ and $\mathbf{R}$, the system is defined by its matrices $\mathbf{F}_\theta$, $\mathbf{G}_\theta$ and $\mathbf{H}_\theta$. The subscript θ indicates that these matrices are functions of a vector of parameters θ: parameter estimation is the process of assessing θ from a set of observations $\mathbf{y}_{1:N} = \{\mathbf{y}_t,\ t \in 1...N\}$. The process noise $\{\mathbf{w}_t\}$ is a way to account for modelling approximations, unrecognized inputs or noise-corrupted input measurements [23]. The main target of this work is to show the importance of this term in the outcome of a parameter estimation problem.

[26] denote stochastic state-space models as *grey-box* models, as opposed to deterministic *white-box* models which do not account for process noise. This definition of grey-box versus white-box models is however not unanimous in the literature: in the present paper, both alternatives will be denoted as *stochastic* or *deterministic*.

### 2.2. Simplified building modelling

As a discrete-time model, Eq. (1) is not the direct expression of physical conservation equations. The present section describes how a simplified building model, written in continuous time, can be translated to this form in order to perform Kalman filtering, inference and forecasting.

This study uses RC models (or *lumped* models) for simplified building modelling. In the absence of non-linear phenomena (aeraulics, long-wave radiation, moisture transfer …), these models can be expressed as linear state space models in continuous time [23]:

$$\dot{\mathbf{T}}(t) = \mathbf{A}_\theta\,\mathbf{T}(t) + \mathbf{B}_\theta\,\mathbf{u}(t) + \mathbf{w}(t) \tag{3}$$

$$\mathbf{y}(t) = \mathbf{C}_\theta\,\mathbf{T}(t) + \mathbf{v}(t) \tag{4}$$

- $\mathbf{T}(t)$ is the $p$-dimensional vector of all temperatures calculated by the model;
- $\mathbf{y}(t)$ is the $q$-dimensional vector of output temperatures that will be compared to measurements (typically $q = 1$ and $\mathbf{y}(t)$ is the indoor temperature);
- $\mathbf{u}(t)$ is the $r$-dimensional vector of boundary conditions: prescribed heat input, solar radiation and outdoor temperature;
- $\mathbf{w}(t) \sim \mathcal{N}(0, \mathbf{Q}_c)$ is the process noise in continuous time;
- $\mathbf{v}(t) \sim \mathcal{N}(0, \mathbf{R}_c)$ is the observation noise in continuous time.

The target of a model calibration problem is to fit the model output $\mathbf{y}(t)$ with measurements carried in an experimental setting, in order to estimate parameter values θ that constitute the terms of the system matrices $\mathbf{A}_\theta$, $\mathbf{B}_\theta$ and $\mathbf{C}_\theta$ (the latter is often a matrix of zeros and ones indicating which of the temperatures $\mathbf{T}(t)$ are observed). Measurements are classified according to their role with respect to the model: inputs $\mathbf{u}_{1:N} = \{\mathbf{u}_t,\ t \in 1...N\}$ are outdoor temperature, solar radiation and heating power; the observed output $\mathbf{y}_{1:N}$ is the indoor temperature. The equations for the specific RC models used in this study will be detailed below in Sec. 3.3.

The stochastic model described by Eq. (3) must be discretized in order to specify its evolution between discrete time coordinates, as in Eq. (1). Let us denote the sample interval length $\Delta t$ and assume that the inputs $\mathbf{u}(t)$ are constant during this interval. Then the system made of Eqs. (3) and (4) can be translated into the discrete system made of Eqs. (1) and (2) through:

$$\mathbf{F}_\theta = \exp(\mathbf{A}_\theta\,\Delta t) \tag{5}$$

$$\mathbf{G}_\theta = \mathbf{A}_\theta^{-1}(\mathbf{F}_\theta - \mathbf{I})\,\mathbf{B}_\theta \tag{6}$$

$$\mathbf{H}_\theta = \mathbf{C}_\theta \tag{7}$$

$$\mathbf{Q} = \int_0^{\Delta t} \exp(\mathbf{A}_\theta\,\Delta t)\,\mathbf{Q}_c\,\exp(\mathbf{A}_\theta^T\,\Delta t)\mathrm{d}t \tag{8}$$

$$\mathbf{R} = \frac{1}{\Delta t}\mathbf{R}_c \tag{9}$$

The typical workflow for calibrating an RC model is to first select a model structure (1R1C, 2R2C, etc.) and write its equations in matrix form 3 and 4. Measurements are then acquired at a sample rate of $\Delta t$ and the model is discretized with Eq. (5) through (9) in order to obtain the system in the form of Eqs. (1) and (2). Once the system is expressed as such, one can proceed to the next steps of filtering, parameter estimation and forecasting.

### 2.3. Kalman filter

Let us first suppose that the parameters θ of the system are known, and a sequence of output observations $\mathbf{y}_{1:N}$ and input variables $\mathbf{u}_{1:N}$ has been obtained.

Most building model calibration practitioners work with deterministic models, in which modelling errors are not explicitly expressed as it is in the previous section. In these circumstances, all states $\mathbf{x}_{1:N}$ of the system are predicted given some parameter values θ, and compared with observations in a single operation. The objective function of the parameter estimation problem is the sum of squared errors $\sum_{t=1}^{N}(\mathbf{y}_t - \mathbf{H}_\theta\mathbf{x}_t)^2$. In a stochastic setting however, the model is considered potentially wrong and its error covariance $\mathbf{Q}$ might also be unknown. If the model is linear, the estimation of the underlying states