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Bayesian inference for estimating thermal properties of a historic building wall

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ABSTRACT

In this paper, the use of Bayesian inference is explored for estimating both the thermal conductivity and the internal convective heat transfer coefficient of an old historic building wall. The room air temperature, as well as the temperatures at the surface and within the wall have been monitored during one year and then used to solve the identification problem. With Bayesian inference, the posterior distributions of the unknown parameters are explored based on their prior distributions and on the likelihood function that models the measurement errors. In this work, the Markov Chain Monte Carlo method is used to explore the posterior distribution. The error of the inadequacy of mathematical model are considered using the approximation error model. The distribution of the estimated parameters have a small standard deviation, which illustrates the accuracy of the method. The parameters have been compared to the standard values from the French thermal regulations. The heat flux at the internal surface has been calculated with the estimated parameters and the standard values. It is shown that the standard values underestimate the heat flux of an order by 10%. This study also illustrates the importance of the preliminary diagnosis of a building with the estimation of the thermal properties of the wall for model calibration.

1. Introduction

The current energy and environmental issues lead to focus on the building sector. Actually, it represents almost 33% of the world global energetic consumption and about 20% of the CO_2 emissions [1]. The stock increases slowly. For instance in France, the average growth is around 1% [2]. Thus, the issues of energy efficiency on existing buildings are primordial. Different tools have been developed since the past decades for the precise assessment of building energy efficiency. The Annex 41 [3] of the International Energy Agency reported on most detailed models and their successful applications for engineering, expertise or research purposes.

However, the modelling simplifications and uncertainties bring to discrepancies between model predictions and real performance [4]. To reduce these discrepancies, one issue is to calibrate the model using on-site measurements combined with model identification methods. The aim is to diminish the uncertainties of the input parameters of the model.

Several works of such experiments at the scale of the wall can be found in literature. Instrumented test cells, as those presented in Refs. [2,5–11] provide measured dataset as temperature and relative humidity at different points within the wall for given boundary conditions. Some experiments at the scale of the whole-building are addressed in Refs. [12,13]. These data can be used to estimate the properties (transport and capacity coefficient) of the materials constituting the walls. For instance, properties of the wall are identified considering heat transfer in Refs. [12–15] and considering coupled heat and moisture transfer in Refs. [16,17]. These studies concern instrumented test cells or recently built buildings and to our knowledge a few studies as those in Refs. [18–20], have been done on the estimation of the wall properties for historic buildings using on-site measurements. This estimation is worth of investigation as these buildings were built in the last century, using traditional techniques and materials available around the construction site. The material properties strongly vary from one site to another and there is a lack of detailed data [2,21,22].

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Numerous works proposing methods to solve identification problems have already been presented in the literature. Readers may refer to [23,24] for a primary overview of the different methods. Among these techniques, the Bayesian framework [25] offers an interesting approach: the solution of the identification problem is considered as a statistical inference. The focus is to analyse the *posterior* distribution corresponding to the probability of the unknown parameters given the measurements. Examples of successful use of this approach can be consulted in Refs. [26–28].

As it is decisive in building energy efficiency, this paper focuses on estimating the wall thermophysical properties of an existing historic building using on-site measurements. The focus is to apply the robust Bayesian technique to estimate the thermal conductivity of the wall and the internal convective heat transfer coefficient. In the next sections, the physical problem and the Bayesian inference are described. The on-site experimental set-ups of the building are also detailed. Then, the Bayesian approach is used for the solution of two inverse problems.

2. Methodology

2.1. Physical problem and mathematical formulation

The physical problem involves transient three-dimensional heat conduction in a domain Ω formed by several regions. The partial domain defining region i is denoted by Ω_i, $\forall i \in \{1,\ldots,I\}$ so that $\Omega = \Omega_1 \cup \Omega_2 \cup \ldots \Omega_I$. The thermophysical properties of each region are supposed constant and the contact between neighbouring regions Ω_i and Ω_j, $\forall i \in \{1,\ldots,I\}$, $\forall j \in \{1,\ldots,I\}, i \neq j$ is assumed to be perfect. The initial temperature in the body is supposed uniform and equal to u_0. The temperature at the surface of the body denoted by Γ_u, is maintained at u_p, while a Robin condition is imposed at the surface of the body denoted by Γ_h. Therefore, the heat conduction problem can be written in its dimensionless form as:

$$\rho_i c_i \frac{\partial u_i}{\partial t} - \mathrm{div}(\kappa_i\, \mathbf{grad}\, u_i) = 0 \quad \text{in } \Omega_i, \quad \text{for } t > 0 \tag{1a}$$

$$-\kappa_i\, \mathbf{grad}\, u_i \cdot \mathbf{n}_i = -\kappa_j\, \mathbf{grad}\, u_j \cdot \mathbf{n}_j \quad \text{at the interface between } \Omega_i \text{ and } \Omega_j, \quad \text{for } t > 0 \tag{1b}$$

$$u_i = u_j \quad \text{at the interface between } \Omega_i \text{ and } \Omega_j, \quad \text{for } t > 0 \tag{1c}$$

$$u_i = u_p \quad \text{at } \Gamma_u, \quad \text{for } t > 0 \tag{1d}$$

$$-\kappa_i\, \mathbf{grad}\, u_i \cdot \mathbf{n}_i = h(u - u_\infty) \quad \text{at } \Gamma_h, \quad \text{for } t > 0 \tag{1e}$$

$$u_i(r,t) = u_{ic}(r) \quad \text{in } \Omega_i, \quad \text{for } t = 0 \tag{1f}$$

where the following quantities are defined as:

$$x = \frac{\overline{x}}{\overline{L}}, \quad u = \frac{\overline{T}}{\overline{T}_{ref}}, \quad h = \frac{\overline{L}\overline{h}}{\overline{\kappa_r}}, \quad t = \frac{\overline{\kappa_r}\overline{t}}{\overline{\rho}_r \overline{c}_r \overline{L}^2}, \quad \kappa = \frac{\overline{\kappa}}{\overline{\kappa_r}} \tag{2a}$$

$$\rho c = \frac{\overline{\rho c}}{\overline{\rho}_r \overline{c}_r} \quad u_p = \frac{\overline{T}_p}{\overline{T}_{ref}}, \quad u_\infty = \frac{\overline{T}_\infty}{\overline{T}_{ref}}, \quad u_{ic} = \frac{\overline{T}_{ic}}{\overline{T}_{ref}} \tag{2b}$$

where $\overline{T}$ is the temperature, $\overline{T}_{ref}$ a reference temperature, $\overline{\kappa_r}$ a reference thermal conductivity, $\overline{\rho c}$ a reference heat capacity per volume, $\overline{T_{ic}}$ the initial temperature of the wall, $\overline{L}$ a characteristic dimension of the wall, $\overline{T}_p$ the prescribed temperature on surface Γ_u, $\overline{T}_\infty$ air temperature corresponding to Γ_h, and $\overline{h}$ the convective coefficient on surface Γ_h.

The problem given by Equation (1) is a direct problem when all the thermophysical properties, initial and boundary conditions, as well as the body geometry are known. The direct problem is solved by using finite differences considering standard discretization and incremental techniques. In this work, an implicit scheme combined with central spatial discretization is used to build the model. Such direct problem solution obtained by using a sufficiently converged grid is denoted by $\mathbf{u}(x,t)$.

2.2. Bayesian technique for inverse problems

The unknown parameters appearing in the formulation of the physical problem given by Equation (1) are here denoted as vector $\mathbf{P} = [P_1, \ldots P_p]$. Let's also assume that transient measurements, represented by the vector $\mathbf{Y}$ are available. The measured data in this work is supposed to be given by temperature values recorded by thermocouples at specific points inside the body or over its surface. The Markov chain Monte Carlo (MCMC) method is applied to estimate the *posterior* distribution of the parameters $\mathbf{P}$, within the Bayesian framework [25,29–31]. Bayes' theorem is stated as [29]:

$$\pi_{posterior}(\mathbf{P}) = \pi(\mathbf{P}|\mathbf{Y}) = \frac{\pi(\mathbf{Y}|\mathbf{P})\pi(\mathbf{P})}{\pi(\mathbf{Y})} \tag{3}$$

where $\pi_{\text{posterior}}(\mathbf{P})$ is the *posterior* probability density, $\pi(\mathbf{P})$ is the *prior* density, $\pi(\mathbf{Y}|\mathbf{P})$ the *likelihood* function and $\pi(\mathbf{Y})$ the marginal probability density of measurements. The computation of $\pi(\mathbf{Y})$ is usually not needed for practical calculations, thus Bayes' theorem is commonly written as:

$$\pi_{posterior}(\mathbf{P}) = \pi(\mathbf{P}|\mathbf{Y}) \propto \pi(\mathbf{Y}|\mathbf{P})\pi(\mathbf{P}) \tag{4}$$

Here, the MCMC algorithm used to explore the *posterior* distribution is that proposed by Metropolis and Hastings [29–34]. The algorithm starts with the selection of a proposal distribution $p(\mathbf{P}^*,\mathbf{P}^{n-1})$ used to draw a new candidate state $\mathbf{P}^*$, given the current state $\mathbf{P}^{n-1}$ of the Markov chain. Once this jumping distribution is selected, the Metropolis-Hastings algorithm can be implemented by repeating the following steps until reaching the total number of states N [29–34]:

1. Consider the state n of the Markov Chain and sample a candidate point $\mathbf{P}^*$ from a proposal distribution, $p(\mathbf{P}^*,\mathbf{P}^{n-1})$;
2. Compute the solution of the direct problem (1) with parameter $\mathbf{P}^*$;
3. Calculate the *posterior* $\pi(\mathbf{P}^*|\mathbf{Y})$ with Eq. (4);
4. Calculate the acceptance factor: