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Markov Chain Monte Carlo (MCMC) approach for the determination of thermal diffusivity using transient fin heat transfer experiments

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ABSTRACT

A simultaneous estimation of fin parameter "m" and thermal diffusivity "α" of a fin material is accomplished by conducting in-house unsteady experiments on a fin of constant area losing heat to still air by natural convection. The material of the fin is mild steel and the surface is highly polished. The fin protrudes from an aluminium base and beneath the aluminium base, a heater is provided to heat the fin. Upon reaching steady state, the power is switched off, transient cooling takes place and the temperature distribution for various time intervals is recorded using a data logger. The temperature varies along the height of the fin and also with respect to time. Bayesian inference is then applied to statistically determine the unknown parameters "m" and thermal diffusivity "α" simultaneously. Markov chain Monte Carlo method (MCMC) is used for sampling the fin parameter "m" and thermal diffusivity "α" of the material. The parameters are retrieved with and without MCMC using a wealth of temperatures generated from experiments with minimum number of time instances. The usefulness of priors in improving the estimates of parameters is investigated. The uncertainity in the form of standard deviation of the parameters estimated, an inherent output of the Bayesian frame work is also reported.

1. Introduction

Inverse heat transfer is a well known topic of interest for researchers seeking to determine thermophysical properties, transport properties and boundary heat flux. The necessity of such an increased interest is to determine the cause for the obtained effect which is not often times so trivial as there can be several causes for the same effect. Inverse problems are encountered in almost every branch of science and engineering. Standard heat transfer problems i.e, direct or forward problems have well posed conditions whereas inverse heat transfer problems are mostly ill posed. For a standard well posed problem, the following condition must be satisfied.

- The solution must exist.
- There must be an unique solution
- The solution must be stable even for small perturbations on the input

The statistical assumptions made regarding the errors as follows.

1. Errors are Gaussian and uncorrelated, as well as the measurements and the parameters are independent.
2. The errors are additive, i.e., $Y_i = T_i + \in_i$.
3. The error $\in_i$ has zero mean, $E(\in_i) = 0$.
4. The errors contains constant variance, $\sigma_i^2 = E[Y_i - E(Y_i)]^2 = \text{constant}$

Inverse problems are generally very sensitive to random errors associated with experiments. If the number of parameters to be retrieved increases, the difficulty in estimating them simultaneously dramatically increases. Fig. 1 shows a general depiction for parameter estimation in inverse problems. Any inverse methodology should contain the forward model wherein the physics of the experiment is modelled invariably as an ordinary/or partial differential equation subject to various boundary conditions and initial conditions. The forward model is solved for guess values of the input state vector x such as thermal conductivity k, specific heat C_p, convective heat transfer coefficient h and emissivity. As far as heat transfer is concerned, the output of a forward model is generally the temperature distribution and this is denoted by Y_{sim}. If Y_{sim} is a vector of the simulated temperatures and Y_{meas} is the data vector

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Nomenclature

A_c	cross sectional area of the fin, m^2
A_s	surface area of the test plate, m^2
Fo	cell Fourier number ($\alpha\Delta t/\Delta x^2$)
h	convective heat transfer coefficient, $Wm^{-2}K^{-1}$
k_f	thermal conductivity of mild steel, $Wm^{-1}K^{-1}$
L	length of the fin, m
Nu	Nusselt number, h L/k_f
p	perimeter of the fin, m
P	power, W
PPDF	posterior probability density function
Q	heat input, W
RMSE	root mean square error
SD	standard deviation of convective heat transfer coefficient ($Wm^{-2}K^{-1}$) or thermal conductivity ($Wm^{-1}K^{-1}$) as the case may be
Std.Er.	standard error
T	temperature, K
T_b	temperature at the base, K
T_∞	ambient temperature, K
v	voltage, Volt

Greek symbols

ΔT	temperature difference, $(T-T_\infty)$, K
μ_p	mean of Gaussian prior for heat transfer coefficient ($Wm^{-2}K^{-1}$) or thermal conductivity ($Wm^{-1}K^{-1}$) as the case may be
Φ	state vector
ϕ	non dimensional temperature, $\dfrac{T-T_\infty}{T_b-T_\infty}$
σ_p	standard deviation of Gaussian prior for heat transfer coefficient $Wm^{-2}K^{-1}$ or thermal conductivity $Wm^{-1}K^{-1}$
θ	temperature excess, $T-T_\infty$, K
ξ	non-dimensional length, x/L

Subscripts

b	base
end	tip
ini	initial
meas	measured temperature distribution
sim	simulated temperature distribution
∞	ambient

obtained from experiments, invariably the goal is to minimise the L2 norm $\sum_{i=1}^{n}(Y_{\text{means},i}-Y_{\text{sim},i})^2$. If the error is acceptable the procedure is stopped, else the next sample of state vector, x is chosen by the inverse method which can be a calculus based method or stochastic based method. This process will continue till the error is minimized to an acceptably low value. There are some versatile and powerful methods to solve inverse problems that are gradient based and a few of these are

- Levenberg–Marquardt algorithm
- Conjugate gradient method
- Conjugate gradient method with adjoint problem estimation can also be done using global search methods like
- Genetic algorithms
- Simulated annealing
- Bayesian inference with incorporation of priors coupled with a suitable sampling strategy (direct minimization of L2 norm not done here)

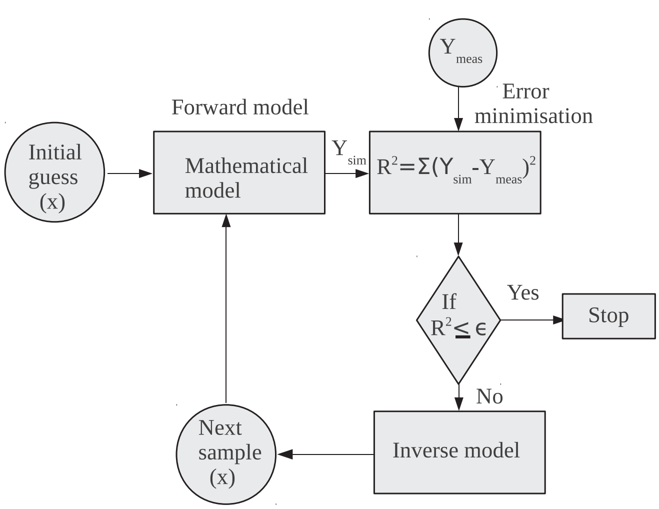

Fig. 1. General depiction for a parameter estimation problem.

Stochastic based methods estimate the mean of the parameters together with the uncertainity (standard deviation) associated with estimates. The problem of converging to local minima is generally encountered in calculus based methods and this can be avoided in stochastic methods. Bayesian inference is a technique based on the Baye's theorem which contains the inherent quality of incorporating prior knowledge about the parameters to be estimated. This offers the greatest advantage in so far as estimation is concerned. The MCMC method within the Bayesian framework of statistics is a powerful methodology for estimation of parameters.

2. Literature survey

Published literature in the area of inverse heat transfer is large. Beck et al. [1] gives a basic understanding of modelling and solving the inverse heat conduction problems. Orlande [2] has given a comprehensive study of inverse problems in the diversified field of heat transfer and has discussed about a number of studies published in this area in recent years. Huang and Yan [3] solved a one dimensional transient conduction problem and estimated the temperature dependent thermal conductivity and specific heat numerically using the conjugate gradient method. Ji et al. [4] used a recursive least-squares algorithm for the estimation of surface heat flux of inverse heat conduction problem using experimental data. Dowding and Blackwell [5] discussed the measurement of thermal properties of solids and emphasized the importance of experimental design. They estimated the thermal conductivity and volumetric heat capacity from two dimensional transient heat conduction. Blackwell et al. [6] estimated the thermal conductivity of 304 stainless steel using measurements and discussed the sensitivity factors associated with it. Monde and Mitsutake [7] estimated thermal diffusivity using an inverse solution for the one dimensional heat conduction equation. The numerical prediction of the estimation was validated with experiments. With the same procedure they simultaneously estimated thermal conductivity and thermal diffusivity numerically. Chen et al. [8] applied a hybrid numerical algorithm of the Laplace transform technique with the least-squares scheme to predict the unknown surface temperature of two-sided boundary conditions for two-dimensional inverse heat conduction problems. Boudenne et al.