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Heat transfer in a liquid film on an unsteady stretching sheet

I.-Chung Liu [a,*], Helge I. Andersson [b]

[a] Department of Civil Engineering and Institute of Earthquake and Disaster Mitigation, National Chi Nan University, 1 University Road, Puli, Nantou, Taiwan 545, ROC

[b] Department of Energy and Process Engineering, Norwegian University of Science and Technology, 7491 Trondheim, Norway

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Abstract

The heat transfer in a liquid film driven by a horizontal sheet is examined. The stretching rate and temperature of the sheet vary with time. The boundary layer equations for momentum and thermal energy are reduced to a set of ordinary differential equations by means of an exact similarity transformation. Numerical solutions of the resulting four-parameter problem are provided. It is observed that the variation of the sheet temperature with distance and with time has analogous effects both on the free surface temperature and the heat transfer rate (Nusselt number) at the sheet.

Keywords: Unsteady stretching sheet; Film flow; Heat transfer; Similarity solutions

1. Introduction

Both the flow and heat transfer in a viscous fluid over a stretching surface have been extensively investigated during the past decades owing to its importance in industrial and engineering applications. Examples are heat treatment of materials manufactured in an extrusion process and a casting process of materials. Cooling of stretching sheets is needed to assure the best quality of the material and requires dedicated control of the temperature and, therefore, knowledge of flow and heat transfer in such systems.

Motivated by the process of polymer extrusion, in which the extrudate emerges from a narrow slit, Crane [1] was the first to examine the *semi-infinite* fluid flow driven by a linearly stretching surface. Later on several authors [2–10] studied various aspects of this problem, such as the heat, mass and momentum transfer in viscous flows with or without suction or blowing through the sheet. These studies all considered the steady flow, heat and mass transfer in a semi-infinite fluid layer driven by a continuous stretching sheet. Wang [11], on the other hand, investigated the hydrodynamic behavior of a *finite* fluid body, i.e. a thin liquid film, driven by an unsteady stretching surface. He introduced a similarity transformation to reduce the time-dependent momentum equation to a third-order non-linear ordinary differential equation (ODE) with an unsteadiness parameter S. For positive values of S, Wang [11] found that there exists no solution of the hydro-mechanical problem if S falls outside the range $[0, 2]$, whereas the dimensionless film thickness is a monotonically decreasing function of S within this parameter interval. He also observed that the thin-film problem reduces to Crane's [1] original problem of a semi-infinite fluid body when the unsteadiness parameter S approaches zero. On the other hand, the film thickness becomes infinitesimal small when S tends to the limit 2. Andersson et al. [12] extended Wang's [11] problem and analyzed the accompanying heat transfer in the liquid film driven by an unsteady stretching surface. They discussed the physical mechanisms that govern the observed thermal characteristics for various Prandtl numbers and different values of the unsteadiness parameter S. More recently, Wang [13] reconsidered exactly the same problem as in Ref. [12] and provided an analytic series solution by means of the homotopy analysis method. The potential influence of thermo-capillarity on the flow and heat transfer was examined by Dandapat et al. [14].

In the present study we aim to generalize the analysis by Andersson et al. [12] of the thermal characteristics of a liquid film driven by an unsteady stretching surface. Here, we consider a

* Corresponding author.
E-mail address: icliu@ncnu.edu.tw (I.-C. Liu).

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more general form of the prescribed temperature variation of the stretching sheet than that considered in Ref. [12]. Exact similarity solutions can nevertheless be achieved. Since closed-form analytical solutions are not readily available, the resulting non-linear ODEs are integrated numerically. Temperature profiles and heat transfer rates at the surface (i.e. Nusselt number) will be presented for representative values of the unsteadiness parameter S, the two sheet-temperature characteristics r and m, and the Prandtl number P.

2. Problem formulation

Let us consider the thin elastic sheet that emerges from a narrow slit at the origin of the Cartesian coordinate system shown in Fig. 1. The continuous sheet aligned with the x-axis at $y = 0$ moves in its own plane with a velocity $U_s(x,t)$ and the temperature distribution $T_s(x,t)$ varies both along the sheet and with time. A thin liquid film with uniform thickness $h(t)$ rests on the horizontal sheet. The governing time-dependent equations for mass, momentum and energy conservation are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and v are the velocity components in the x and y directions, respectively, and T is the temperature. We assume that all fluid properties are constant. Here, ρ is the density, μ is the dynamic viscosity and k is the thermal conductivity of the incompressible fluid. Thus, the kinematic viscosity is $\nu = \mu/\rho$ and the thermal diffusivity is $\kappa = k/\rho c_p$ where c_p is the heat capacity at constant pressure.

In the derivation of the governing equations (1)–(3), the conventional boundary layer approximation has been invoked. This is justified by the assumption that the film thickness h is substantially smaller than a characteristic length scale L in the direction along the sheet. Mass conservation (1) then implies that the ratio v/u between the two velocity components is of the order $h/L \ll 1$. Similarly, streamwise diffusion of momentum and thermal energy is of the order $(h/L)^2$ smaller than the corresponding diffusion perpendicular to the sheet. For this reason, the streamwise diffusion terms have been neglected in Eqs. (2) and (3). The mathematical character of the partial differential equations is thereby changed from elliptic to parabolic, which in turn effects the number of boundary conditions required. The appropriate boundary conditions for the above boundary layer equations are

$$u = U_s, \quad v = 0, \quad T = T_s \quad \text{at } y = 0 \tag{4}$$

$$\frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0, \quad v = \frac{\mathrm{d}h}{\mathrm{d}t} \quad \text{at } y = h(t) \tag{5}$$

where $h(t)$ is the free surface elevation of the liquid film, i.e. the film thickness. The first parts of Eq. (5) reflect the absence of viscous shear stress and heat flux at the free surface, while the last part is a kinematic free-surface condition.

The fluid motion within the liquid film is caused only by the viscous shear arising from the stretching of the elastic sheet. The stretching velocity $U_s(x,t)$ is assumed to be of the same form as that considered by Wang [11], Andersson et al. [12] and Wang [13]:

$$U_s = \frac{bx}{1 - ct} \tag{6}$$

where both b and c are positive constants with dimension reciprocal time. Here, b is the *initial* stretching rate, whereas the *effective* stretching rate $b/(1 - ct)$ is increasing with time. In the context of polymer extrusion the material properties and in particular the elasticity of the extruded sheet may vary with time even though the sheet is being pulled by a constant force. The dimensionless ratio $S \equiv c/b$ eventually became the only parameter in Wang's analysis, which in the limit $S \to 0$ reduces to the steady-state problem due to Crane [1]. It should be emphasized that the initial stretching rate b does not represent an externally imposed time scale, as incorrectly argued by Vleggaar [3] and Kumari et al. [9]. With unsteady stretching (i.e. $c \neq 0$), however, c^{-1} becomes the representative time scale of the resulting unsteady boundary layer problem. The adopted formulation of the velocity sheet velocity $U_s(x,t)$ in Eq. (6) is valid only for times $t < c^{-1}$ unless $c = 0$.

The temperature of the surface of the elastic sheet is similarly assumed to vary both along the sheet and with time, in accordance with:

$$T_s = T_0 - T_{\mathrm{ref}}\frac{dx^r}{\nu}(1 - ct)^{-m} \tag{7}$$

Here, T_0 is the fixed slit temperature at $x = 0$ (except for $r = 0$) and T_{ref} is a reference temperature which will be taken as $T_{\mathrm{ref}} = T_0$ in the present study. The constant of proportionality d is assumed to be positive with dimension (length^{2-r} time^{-1}). The power indices r and m enable us to examine a variety of different temperature variations. With $r > 0$, the sheet temperature decreases as x^r with the distance from the slit. Similarly, for $m > 0$ the sheet temperature at a fixed location x is reduced with time in proportion to $(1 - ct)^{-m}$. The rather general temperature variation (7) represents a generalization of the sheet temperature considered by Andersson et al. [12] and Wang [13]

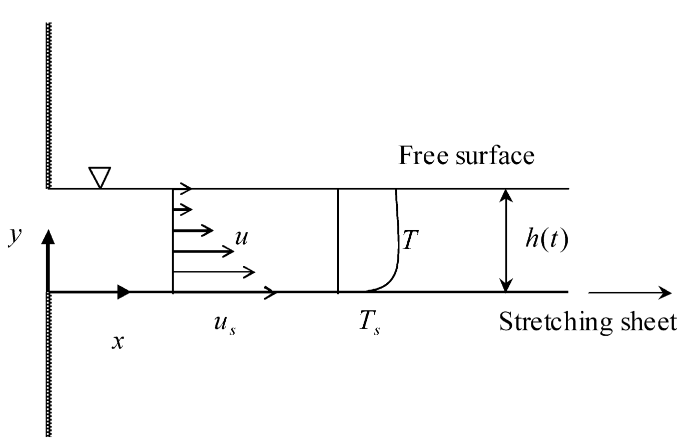

Fig. 1. Physical configuration and coordinate system.