International Journal of Thermal Sciences 50 (2011) 2256–2263

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

An analytical solution for boundary layer flow of a nanofluid past a stretching sheet

M. Hassani [a,*], M. Mohammad Tabar [b], H. Nemati [a], G. Domairry [a], F. Noori [a]

[a] Faculty of Mechanical Engineering, Babol Noshirvani University of Technology, Babol, P.O. Box 484, Iran
[b] Department of Mechanical Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran

A R T I C L E I N F O

Article history:
Received 18 December 2010
Received in revised form
14 May 2011
Accepted 21 May 2011
Available online 25 June 2011

Keywords:
Nanofluid
Stretching sheet
Homotopy analysis method
Brownian motion
Thermophoresis

A B S T R A C T

In this paper, the problem of boundary layer flow of a nanofluid past a stretching sheet has been investigated analytically by using the Homotopy Analysis Method. Both the effects of Brownian motion and thermophoresis are considered simultaneously. An analytical solution is presented which depends on the Prandtl number Pr, Lewis number Le, Brownian motion number Nb and thermophoresis number Nt. The results show that the reduced Nusselt number is a decreasing function of each dimensionless number, while the reduced Sherwood number is an increasing function of higher Pr and a decreasing function of lower Pr number for each Le, Nb and Nt numbers like the results presented by Khan and Pop. Contrary the results presented by Khan and Pop, It is found that the reduced Nusselt number decreases with the increase in Pr for many Nb numbers. However for a special Nb, there are conversely interesting results that are clearly discussed in this paper.

1. Introduction

Most problems and scientific phenomena such as heat transfer and diffusion ones function nonlinearly. We know that except a limited number of these problems, most of them do not have analytical solutions. So these nonlinear equations should be solved using numerical methods or other analytical methods. There are some restrictions to solve this problem, first we encountered with the nonlinearity of system, and on the other hand, this problem is a boundary value problem with infinite boundary values. In this study Homotopy Analysis Method (HAM) which was expected by Liao [1–3] has been successfully applied as an analytical method to solve the nonlinear problem. This method has been successfully applied to solve many types of nonlinear problems [4–9]. The convergence of the series solution is also explicitly discussed. The flow over a stretching surface has been utilized in many engineering processes with applications in industries such as extrusion, melt-pinning, the hot rolling, wire drawing, glass fiber production, manufacture of plastic and rubber sheet cooling of a large metallic plate in a bath, which may be an electrolyte, etc. Experimental results show that the velocity of the stretching surface is approximately proportional to the distance from the orifice [10]. Crane [11] studied the steady two-dimensional in compressible boundary layer flow of a Newtonian fluid caused by the stretching of an elastic flat sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point due to the application of a uniform stress. Crane [11] obtained an exact solution of the two-dimensional Navier–Stokes equations. After this pioneering work, the flow field over a stretching surface has drawn considerable attention and a good amount of literature has been generated on this problem [12–17]. In recent years, some interest has been given to the study of convective transport of nanofluids. Nano-scale particle added fluids are called as nanofluid, which is firstly utilized by Choi [18]. Choi et al. [19] showed that the addition of a small amount (less than 1% by volume) of nano particles to conventional heat transfer liquids increased the thermal conductivity of the fluid up to approximately two times. Nanotechnology aims to manipulate the structure of the matter at the molecular level with the goal for innovation in virtually every industry and public endeavor including biological sciences, physical sciences, electronics cooling, transportation, the environment and national security. Some numerical and experimental studies on nanofluids include thermal conductivity [20], convective heat transfer [21–25]. Buongiorno [26] and Kakaç and Pramuanjaroenkij [27] have investigated a comprehensive survey of convective transporting nanofluids. Khan and pop [28] analyzed the development of

* Corresponding author.
E-mail address: meghzip@gmail.com (M. Hassani).

1290-0729/$ – see front matter © 2011 Elsevier Masson SAS. All rights reserved.
doi:10.1016/j.ijthermalsci.2011.05.015

Nomenclature

C	nanoparticle volume fraction
C_∞	ambient nanoparticle volume fraction
C_w	nanoparticle volume fraction at the stretching surface
D_B	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
$f(\eta)$	dimensionless stream function
k	thermal conductivity
Le	Lewis number
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nu	Nusselt number
T	fluid temperature
P	pressure
Pr	Prandtl number
q_m	wall mass flux
q_w	wall heat flux
Re_x	local Reynolds number
Sh_x	local Sherwood number
T_w	temperature at the stretching surface
T_∞	ambient temperature
u, v	velocity components along x- and y-axes
u_w	velocity of the stretching sheet
x, y	Cartesian coordinate

Greek symbols

α	thermal diffusivity
$\phi(\eta)$	rescaled nanoparticle volume fraction
η	similarity variable
$\theta(\eta)$	dimensionless temperature
υ	kinematic viscosity of the fluid
ρ_f	fluid density
ρ_p	nanoparticle mass density
$(\rho C)_f$	heat capacity of the fluid
$(\rho C)_p$	effective heat capacity of the nanoparticle
τ	ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid
ψ	stream function

the steady boundary layer flow, heat transfer and nanoparticle fraction over a stretching surface in a nanofluid. Their solution depended on a Prandtl number Pr, a Lewis number Le, a Brownian motion number Nb and a thermophoresis is number Nt. The dependency of the local Nusselt and local Sherwood numbers on these four parameters was numerically investigated. Bachok et al. [29] investigated the steady boundary-layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream. Numerical results were obtained for the skin-friction coefficient, the local Nusselt number and the local Sherwood number as well as the velocity, temperature and the nanoparticle volume fraction profiles for some values of the governing parameters, namely, the plate velocity parameter, Prandtl number, Lewis number, the Brownian motion parameter and the thermophoresis parameter. Recently,some problems about nanofluids are studied by other investigators [30–32].

The objective of the present study is to analyze the development of the steady boundary layer flow, heat transfer and nanoparticle fraction analytically by using the Homotopy Analysis Method over a stretching surface in a nanofluid. This problem is solved numerically by Khan and Pop [28]. This solution depends on a Prandtl number Pr, a Lewis number Le, a Brownian motion number Nb and a thermophoresis number Nt. The dependency of the local Nusselt and local Sherwood numbers on these four parameters is analytically investigated. The results are discussed more in detail and compared with the results presented by Khan and Pop [28] in some cases. The dependency of the results to parameters mentioned above has been shown clearly by more figures than the results parented by Khan and Pop [28].

2. Mathematical model

The steady two-dimensional boundary layer flow of a nanofluid past a stretching surface is considered with the linear velocity $u_w(x) = ax$, where a is a constant and x is the coordinate surface with the linear velocity measured along the stretching surface, as shown in Fig. 1. The flow takes place at $y \geq 0$, where y is the coordinate measured normal to the stretching surface. The basic steady conservation of mass, momentum, thermal energy and nanoparticles equations for nanofluids can be written in Cartesian coordinates x and y as, see Khan and Pop [28].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial P}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_f}\frac{\partial P}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau\left\{D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right]\right\} \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

Subject to the boundary conditions

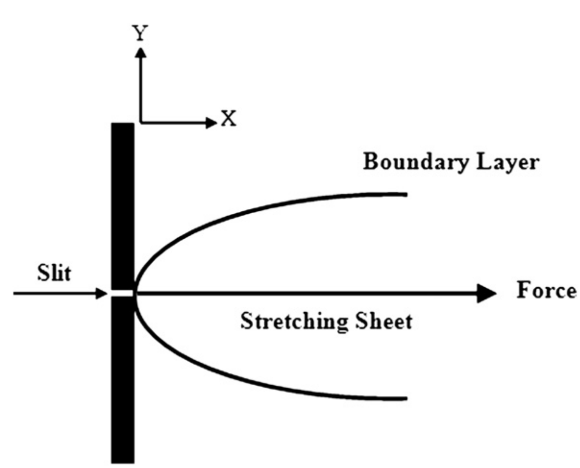

Fig. 1. Physical model and coordinate system.