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Multiple solutions of heat and mass transfer of MHD slip flow for the viscoelastic fluid over a stretching sheet

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ABSTRACT

In this paper we investigate the magnetohydrodynamic slip flow of an electrically conducting, viscoelastic fluid past a stretching surface. The main concern is to analytically investigate the structure of the solutions and determine the thresholds beyond which multiple solutions exist or the physical pure exponential type solution ceases to exist. In the case of the presence of multiple solutions, closed-form formulae for the boundary layer equations of the flow are presented for two classes of viscoelastic fluid, namely, the second-grade and Walter's liquid B fluids. Heat transfer analysis is also carried out for two general types of boundary heating processes, either by a prescribed quadratic power-law surface temperature or by a prescribed quadratic power-law surface heat flux. The flow field is affected by the presence of physical parameters, such as slip, viscoelasticity, magnetic and suction/injection parameters, whereas the temperature field is additionally affected by thermal radiation, heat source/sink, Prandtl and Eckert numbers. The regions of existence or non-existence of unique/multiple solutions sketched by the combination of these parameters are initially worked out by providing critical values and then velocity/temperature profiles and skin friction coefficient/Nusselt number are examined and discussed.

1. Introduction

Due to many important applications in engineering processes, the research on boundary layer behavior of a viscoelastic fluid over a continuously stretching surface keeps going. Momentum and heat transfer in a viscoelastic boundary layer over a linear stretching sheet have thus been studied extensively in the recent years because of its ever-increasing usage in polymer processing industry, in particular, in manufacturing process of artificial film, artificial fibers, polymer extrusion, drawing of plastic films and wires, glass fiber and paper production, manufacture of foods, crystal growing, liquid films in condensation process, etc.

In recent years a great deal of work has been carried out to reveal the flow in viscoelastic fluid flow past a stretching surface. Rajagopal et al. [1] studied viscoelastic second-order fluid flow over a stretching sheet by solving the momentum boundary layer equation numerically. Troy et al. [2] discussed uniqueness of the momentum boundary layer equation. Subsequently, Chang [3] and Rao [4] showed the non-uniqueness of the solution and derived different forms of non-unique solution. Nataraja et al. [5] presented the coefficients of skin friction and heat transfer obtained from the closed-form solutions for the boundary layer equations of the flow of Walter's liquid B over a stretching surface. Vajravelu and Roper [6] explored the flow and heat transfer in a viscoelastic fluid over a stretching sheet with power-law surface temperature, including the effects of viscous dissipation, internal heat generation or absorption, and work due to deformation in the energy equation and analyzed the salient features of the flow and heat transfer characteristics. Khan and Sanjayan [7] presented approximate analytical solution of the viscoelastic boundary layer flow over an exponential stretching continuous sheet. Akyildiz and Vajravelu [8] studied the flow of a viscoelastic fluid immersed in a porous medium over a stretching sheet with magnetohydrodynamic flow conditions, by obtaining exact, analytical, and numerical solutions with existence results for the resulting coupled non-linear differential equations. Mushtaq et al. [9] examined the effects of thermal buoyancy on viscoelastic flow of a second-grade fluid past a vertical, continuously stretching sheet. Numerical solutions for the coupled non-linear partial differential are generated by using the local non-similarity method and Keller-Box scheme. Very

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Nomenclature

Roman symbols

a	positive stretching velocity
(a_1, a_2)	heat constants
B	uniform external magnetic field
c	a constant
c_p	specific heat constant
C_f	skin friction coefficient
Ec	Eckert number
f	dimensionless self-similar velocity
k	thermal conductivity
l	dimensional slip parameter
L	dimensionless slip parameter
M	magnetic interaction parameter
Nu	Nusselt number
p	pressure
P	self-similar pressure
Pr	Prandtl number
q	rate of volumetric heat generation/absorption
q_r	radiative heat flux
q_w	heat flux
R	thermal radiation parameter
s	dimensionless suction or injection parameter
t	a scale
T	temperature
T_w	surface temperature
T_∞	free-stream temperature
u	velocity component in x-direction
v	velocity component in y-direction
v_w	dimensional wall mass transfer velocity
(x, y)	longitudinal and transverse directions
z	asymptotic value of the Nusselt numbers

Greek symbols

α_R	mean absorption coefficient
χ	internal heat generation/absorption parameter
η	a scaled boundary layer coordinate
θ	a scaled temperature
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
σ	electrical conductivity
σ_1	Stefan–Boltzmann constant
ϕ	a scaled temperature
λ	exponential constant

Subscripts

w	quantities at wall
∞	quantities at the free-stream

recently, an investigation was conducted by Arnold et al. [10] on the viscoelastic (Walter's liquid B model) fluid flow over a stretching sheet.

The properties of the final product are known to depend greatly on the rate of cooling involved in manufacturing processes. It would be beneficial to have a controlled cooling system for these processes. An electrically conducting polymeric liquid seems to be a good candidate for some industrial applications such as in polymer technology and metallurgy because the flow can be regulated by external means through a magnetic field. The applied magnetic field may play an important role in controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet. In particular, many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. Another interesting application of hydromagnetics to metallurgy lies in the purification of molten metals from non-metallic inclusions by the application of a magnetic field. Bearing this in mind, many authors endeavored to explore the effect of transverse magnetic field on boundary layer flow and heat transfer for Newtonian and non-Newtonian fluids past stretching surfaces. The combined effects of Joule heating and viscous dissipation on the momentum and thermal transport were examined very recently by Chen [11] for the MHD Newtonian fluid flow over a stretching sheet. In the analysis of [12] the effects of thermal radiation and temperature-dependent thermal conductivity on MHD viscoelastic flow were examined. Liu [13] presented analytic solutions for the flow in an electrically conducting, second-grade fluid subject to a transverse magnetic field past a stretching sheet with power-law surface temperature or power-law surface heat flux. Siddheshwar and Mahabaleswar [14] explored the effect of radiation and temperature-dependent heat source on the MHD viscoelastic flow and convective heat transfer over a stretching sheet. Abbas et al. [15] carried out an analysis to study the unsteady MHD two-dimensional boundary layer flow of a second-grade viscoelastic fluid over an oscillatory stretching surface. The flow is induced due to an infinite elastic sheet which is stretched back and forth in its own plane. An analysis was performed by Singh [16] to study heat source and radiation effects on two-dimensional steady flow of an electrically conducting, viscoelastic fluid (Walter's liquid B model) over a stretching sheet in the presence of transverse uniform magnetic field. A magnetic hydrodynamic incompressible viscoelastic fluid over a stretching sheet with electric and magnetic dissipation and non-uniform heat source/sink was recently studied by Hsiao [17].

In most of these investigations no-slip condition is used. In the recent years, micro-scale fluid dynamics in the Micro-Electro-Mechanical Systems (MEMS) received much attention in research. Because of the micro-scale dimensions, the fluid flow behavior belongs to the slip flow regime and greatly differs from the traditional flow [18]. In the situation when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions, see Yoshimura [19], the no-slip condition is inadequate. In such cases the suitable boundary condition is the partial slip. In spite of its importance in polymer and electrochemical industry, no proper attention has been given to the flow analysis with partial slip condition. Wang [20] discussed the partial slip effects on the planar stretching flow. He obtained the perturbation and numerical solutions. Recently Wang [21] applied the slip condition to the linearly stretching Newtonian flow.

It is now well-known that multiple solutions of exponential or trigonometric type exist for the viscoelastic fluid flow over a stretching sheet in the absence of slip, among which the physically acceptable solution is the exponential type. However, no a satisfactory attempt has yet been made to determine the bounds where the physical exponential type solutions terminate or appear multiply. Therefore, the aforementioned works without a prior justification assumed that such kind of solution is simply unique. However, for various physically meaningful values of the