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Non-Darcy mixed convection in a vertical pipe filled with porous medium

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A R T I C L E I N F O

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A B S T R A C T

The present paper reports an analytical as well as numerical investigation of fully developed mixed convective flow in a vertical pipe filled with porous medium. The motion in the pipe is induced by external pressure gradient and buoyancy force. The non-Darcy Brinkman–Forchheimer-extended model has been considered. The Chebyshev spectral-collocation method has been used to solve the coupled differential equations numerically. A comprehensive investigation on the dependency of mixed convective flow on governing parameters indicates that depending on the values of other parameters the velocity profile possess point of inflection beyond a threshold value of Rayleigh number (*Ra*). In the case of buoyancy-opposed flow, the velocity profile may contain point of inflection in the center zone and point of separation at the vicinity of the wall. The appearance of point of separation causes the back flow near the wall. In contrast to buoyancy-opposed case, where enhancement of *Ra* increases the magnitude of the center velocity as well as temperature till the distortion appears on both profiles, in buoyancy-assisted case both velocity as well as temperature decrease on increasing *Ra* at the center. The points of separation as well as inflection die out on reducing the media permeability or increasing magnitude of the form drag coefficient. Further, it was observed that for buoyancy-opposed flow the velocity as well as temperature profiles have a kind of distortion beyond a threshold value of *Ra*, which is also a function of other governing parameters. In this situation, the heat transfer rate varies abruptly as a function of *Ra*.

1. Introduction

The research works, in the area of convective heat transfer in fluid saturated porous media, have substantially increased during recent years due to its numerous practical applications encountered in engineering and sciences. Among these works natural and forced convection studies occupy the majority of investigations. The inter-facial area of mixed convection which connects natural and forced convection, in comparison, has not been given due attention in porous media. Mixed convection problems in porous media occur very often in the nature, e.g., in studies of shallow-water and deep-sea hydrodynamics. One important example of mixed convection in shallow-water seas is given by hydrothermal vents by which hot, mineral-rich water ejects through a permeable sea-bed [1]. This problem constitutes a very new research area, and theoretical investigation of it has been largely overlooked. For fluid environments, however, there are many papers which deal with mixed convection, for example, in connection with fluid flow in a vertical pipe [2–8], in a vertical annulus [9,10], and in a vertical channel [11,12].

Few investigations in wall bounded mixed convection through vertical annuli (e.g., [13,14]) and channels (e.g., [15–22]) filled with porous medium are reported. Parang and Keyhani [13] studied the fully developed buoyancy-added mixed convective flow in a vertical annulus employing Darcy–Brinkman model. They found that the Brinkman term has a negligible effect on the flow when Darcy number (Da) is very small. According to Muralidhar [14], in the case of buoyancy-assisted mixed convection, the Nusselt number (*Nu*) increases with increasing Rayleigh number (*Ra*).

In the vertical channel, when the walls are heated uniformly, Hadim [15] investigated the evolution of mixed convection in the entrance region for Darcy as well as non-Darcy cases. In the same year, Hadim along with Chen [16] have reported the extension of the above study for asymmetric heating of the walls. They have studied the effect of media permeability on the buoyancy-assisted mixed convection in the entrance region of a vertical channel at fixed values of Reynolds number, Forchheimer number, Prandtl number, and Grashof number. It was found that distortions in the velocity profile lead to increased heat transfer rates when Da is decreased. While studying the same with discrete heat sources at the walls, in place of asymmetric wall heating, they [17] found, the location of the flow separation from the cold wall does not change while reattachment moves further downstream. The Nusselt number increases with decreasing Da and the effect of Da is more

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pronounced over the first heat source and in non-Darcy regimes. A comprehensive review of laminar wall bounded forced or mixed convection may be found in the book of Nield and Bejan [23]. As indicated by Chen et al. [18] for non-Darcy mixed convection in a vertical channel filled with a porous medium, the buoyancy force can significantly affects *Nu* for higher values of *Ra* and Da and lower values of Forchheimer number. An extensive study of mixed convective flow and its stability in vertical channel filled with porous medium has been reported elsewhere [19–22]. They have shown that the fully developed one dimensional mixed convective flow in the vertical channel does not remain one dimensional always.

It is then natural to ask how these flow dynamics will be modulated when vertical channel is replaced by vertical pipe. To the best knowledge of us, except the work of Chang and Chang [24] in which mixed convection in a vertical tube partially filled with porous medium is reported, mixed convection in vertical pipe filled with a porous medium has not been considered yet. Therefore, an attempt has been made in this direction by investigating the flow dynamics of wall bounded fully developed mixed convection and its dependency on different controlling porous-media parameters. The aim of the article is precisely this.

An outline of the paper is as follows. In Section 2, the mathematical formulation and solution of the physical problem are given. Results and discussions along with validation of the numerical results are reported in Section 3. Finally, some important features of the analysis are concluded in Section 4.

2. Mathematical formulation

We consider a fully developed mixed convection flow caused by an external pressure gradient and a buoyancy force in a semi-infinite vertical pipe [8], filled with porous medium (see Fig. 1). The wall temperature is linearly varying with z^* as $T_w = T_0 + C_1 R_0 z^*$, where C_1 is a constant and T_0 is upstream reference wall temperature and R_0 is radius of the pipe. The gravitational force is aligned in the negative z^*-direction.

The thermo-physical properties of the fluid are assumed to be constant except for density dependency of the buoyancy term in the momentum equation. The porous medium is saturated with a fluid that is in local thermodynamic equilibrium with the solid matrix. The medium is assumed to be isotropic in permeability as well as thermal diffusivity. In expressing the equations for the flow in the porous medium, it should be noted that the Darcy model presents a linear relationship between velocity of discharge and the pressure gradient. As the Darcy model does not hold when the flow velocity is not sufficiently small, or when the permeability is high [25–32], extensions to this model known as Brinkman-extended or Forchheimer-extended models exist [23,29]. In short, the Brinkman term is found to be needed for satisfying a no-slip boundary condition at solid walls, whereas the Forchheimer term accounts for the form drag. Also in analogy with the Navier–Stokes equations, the Darcy model has been extended by including the material derivative. The necessity of the simultaneous inclusion of all or some of these extensions has been discussed in the literature [28,30,33]. The objective of the paper is to understand the fluid flow as well as heat transfer mechanism of the steady, unidirectional fully developed flow (basic flow). Therefore, it is assumed that flow is in vertical direction only i.e. the velocity vector is $(0,0,w^*)$. From the continuity equation, it is clear that w^* is function of r^* only. As a consequence of this, the governing differential equations for momentum and energy, in cylindrical coordinate, of the basic flow can be written as

$$\rho_f \frac{c_F}{K^{1/2}} \left|w^*\right| w^* = -\frac{\mathrm{d}p^*}{\mathrm{d}z^*} + \tilde{\mu}\left[\frac{\mathrm{d}^2 w^*}{\mathrm{d}r^{*2}} + \frac{1}{r^*}\frac{\mathrm{d}w^*}{\mathrm{d}r^*}\right] - \frac{\mu_f}{K} w^* + \rho_f g \beta_T \left(T^* - T_w\right) \tag{1}$$

$$w^* \frac{\partial T^*}{\partial z^*} = \alpha\left[\frac{\partial^2 T^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial T^*}{\partial r^*}\right] \tag{2}$$

In the above equations, w^*, K, p^*, t^*, T^*, and ρ_f are flow velocity, permeability, pressure, time, temperature, and fluid density respectively. Further, g, ϵ, c_F, $\tilde{\mu}$, β_T, and σ are the gravitational acceleration, porosity, form drag coefficient, effective viscosity, volumetric thermal expansion coefficient, and ratio of heat capacities respectively.

Using the following non-dimensional quantities:

$$r = \frac{r^*}{R_0}, \quad W_0 = \frac{w^*}{W_c^*}, \quad z = \frac{z^*}{R_0},$$

$$p = \frac{p^*}{\rho_f W_c^{*\,2}}, \quad \Theta_0 = \frac{T_w - T^*}{C_1 R_0 Re Pr}$$

the dimensionless momentum and energy equations are

$$\frac{\mathrm{d}^2 W_0}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}W_0}{\mathrm{d}r} - \frac{\Lambda}{Da} W_0 - Ra\Theta_0 - Re\frac{\mathrm{d}p}{\mathrm{d}z} - ReF|W_0|W_0 = 0 \tag{3}$$

$$\frac{\mathrm{d}^2 \Theta_0}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\Theta_0}{\mathrm{d}r} = -W_0. \tag{4}$$

The corresponding boundary conditions are given by

$$\frac{\mathrm{d}W_0}{\mathrm{d}r} = \frac{\mathrm{d}\Theta_0}{\mathrm{d}r} = 0 \quad \text{at} \quad r = 0, \tag{5}$$

$$W_0 = \Theta_0 = 0 \quad \text{at} \quad r = 1, \tag{6}$$

with W_0 and Θ_0 being the basic velocity and temperature, respectively.

In the above equations the dimensionless parameters are the Rayleigh number (Ra), Forchheimer number (F), Darcy number

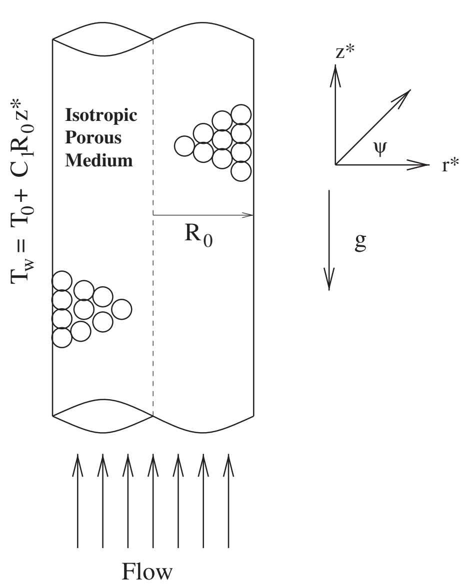

Fig. 1. Schematic of the dimensional physical problem.