International Journal of Thermal Sciences 50 (2011) 749–759

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

A two-dimensional generalized electro-magneto-thermoviscoelastic problem for a half-space with diffusion

Sunita Deswal, Kapil Kalkal*

Department of Mathematics, G. J. University of Science and Technology, Hisar, 125001 Haryana, India

A R T I C L E I N F O

Article history:
Received 18 August 2010
Received in revised form
12 November 2010
Accepted 18 November 2010
Available online 24 December 2010

Keywords:
Electro-magneto-thermoviscoelasticity
G–L theory
Generalized thermoelastic diffusion
Normal mode analysis

A B S T R A C T

The present paper is aimed at studying the effects of viscosity and diffusion on thermoelastic interactions in an isotropic, thermally and electrically conducting half-space solid whose surface is subjected to mechanical and thermal loads. The formulation is applied to the generalized thermoelasticity based on the Green and Lindsay (G–L) theory, where there is an initial magnetic field parallel to the bounding plane. The normal mode analysis is used to obtain the expressions for the variables considered. Numerical computations are performed for a specific material and the results obtained are represented graphically. Comparisons are made within the theory in the presence and absence of viscosity and diffusion.

1. Introduction

The classical theory of thermoelasticity has been generalized and modified into various thermoelastic models that run under the label of "hyperbolic thermoelasticity". The notation hyperbolic reflects the fact that thermal waves are modelled, avoiding the physical paradox of the infinite propagation speed of the classical model. At present, there are several theories of the hyperbolic thermoelasticity. The first was developed by Lord and Shulman [1] who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law. This new law contains the heat flux vector as well as its time derivative. It also contains a new constant that acts as a relaxation time. The second was developed by Green and Lindsay [2]. This theory contains two constants that act as relaxation times and modifies all the equations of the coupled theory, not the heat conduction equation only. Both of these theories ensure finite speeds of propagation for heat wave.

Investigation of the dynamic problem concerning the interactions among electromagnetic field, temperature, stress and strain in a thermoelastic solid is immensely important because of its extensive uses in diverse fields, such as geophysics for understanding the effect of the earth magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices, electrical power engineering, optics etc. Many authors have considered the propagation of electromagneto-thermoelastic waves in an electrically and thermally conducting solid. Nayfeh and Nemat-Nasser [3] studied the propagation of plane waves in a solid under the influence of an electromagnetic field. Sherief and Helmy in [4] examined a two-dimensional problem for a half-space in a magneto-thermoelastic medium. Tianhu et al. [5] considered the electromagnetic-thermoelastic interactions in a semi-infinite perfectly conducting solid subjected to a thermal shock on its surface when the solid and its adjoining vacuum is subjected to a uniform axial magnetic field. Youssef [6] has solved the problem of magneto generalized thermoelasticity by taking the electrical conductivity, thermal conductivity and modulus of elasticity to be variable. Reflection of magneto-thermoelastic waves in a rotating medium has been investigated by Othman and Song [7]. Recently, He and Cao [8] studied the magneto-thermoelastic problem of a thin slim strip placed in a magnetic field and subjected to a moving plane of heat source.

Diffusion can be defined as the spontaneous migration of substances from regions of high concentration to regions of low concentration. There is now a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industrial applications. Thermodiffusion in the solids is one of the transport processes that has great practical importance. Thermodiffusion in an elastic solid is due to the coupling of the fields of temperature, mass diffusion and that of strain. Nowacki [9–11] developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used which implies infinite speeds of propagation of thermoelastic waves. Sherief et al. [12] developed the theory of

* Corresponding author.
E-mail addresses: spannu_gju@yahoo.com (S. Deswal), kapilkalkal_gju@rediffmail.com (K. Kalkal).

1290-0729/$ – see front matter © 2010 Elsevier Masson SAS. All rights reserved.
doi:10.1016/j.ijthermalsci.2010.11.016

Nomenclature

σ_{ij}	components of stress tensor
λ^*	$\lambda_e(1+\alpha_0\frac{\partial}{\partial t})$
μ^*	$\mu_e(1+\alpha_1\frac{\partial}{\partial t})$
$\lambda_e,\ \mu_e$	Lame's constants
$\alpha_0,\ \alpha_1$	viscoelastic relaxation times
β_1^*	$\beta_{1e}(1+\beta_1\frac{\partial}{\partial t})$
β_2^*	$\beta_{2e}(1+\beta_2\frac{\partial}{\partial t})$
β_{1e}	$(3\lambda_e+2\mu_e)\alpha_t$
β_1	$(3\lambda_e\alpha_0+2\mu_e\alpha_1)\frac{\alpha_t}{\beta_{1e}}$
β_{2e}	$(3\lambda_e+2\mu_e)\alpha_C$
β_2	$(3\lambda_e\alpha_0+2\mu_e\alpha_1)\frac{\alpha_c}{\beta_{2e}}$
α_t	coefficient of linear thermal expansion
α_c	coefficient of linear diffusion expansion
θ	$T-T_0$
T	absolute temperature
T_0	temperature of the medium in its natural state assumed to be $\lvert\theta/T_0\rvert \ll 1$
u_i	components of the displacement vector
ρ	density of the medium
e_{ij}	components of the strain tensor
e_{kk}	e cubical dilatation
c	$C-C_0$
C	non-equilibrium concentration
C_0	mass concentration at natural state
c_E	specific heat at constant strain
k	thermal conductivity
D	thermodiffusion constant
τ_0,τ_1	thermal relaxation times
τ^0,τ^1	diffusion relaxation times
μ_0	magnetic permeability
ε_0	electric permeability
F_i	Lorentz force
a	measure of thermodiffusion effect
b	measure of diffusive effect

generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves. Sherief and Saleh [13] worked on a problem of a thermoelastic half-space with a permeating substance, in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion. Recently, Xia et al. [14] studied the dynamic response of an infinite body with a cylindrical cavity whose surface suffers thermal shock using finite element method.

The observed attenuation of the seismic waves in the earth, an important source of information regarding the composition and state of the deep interior can not be explained by assuming the earth to be an elastic solid. Keeping this fact in mind, several problems on wave propagation in a linear viscoelastic solid have been discussed by many research workers. Also with the rapid development of polymer science and plastic industry, as well as the wide use of materials under high temperature in modern technology, the theoretical study and application in viscoelastic materials has become an important task for solid mechanics. The theory of thermoviscoelasticity and the solutions of some boundary value problems of thermoviscoelasticity are investigated by Ilioushin and Pobedria [15]. The works of Tanner [16] and Huilgol and Phan-Thien [17] have made great strides in the last decade in finding solutions for boundary value problems for linear viscoelastic materials including temperature variations in both quasi static and dynamic problems. Due to mathematical difficulties, few attempts have been made to solve two-dimensional problems in generalized electro-magneto-thermo-viscoelasticity in Green and Lindsay theory. Ezzat et al. [18] introduced the state-space approach for the two-dimensional model of generalized thermoviscoelasticity with two relaxation times. Ezzat and El-Karamany [19] established the uniqueness and reciprocity theorems for generalized thermoviscoelasticity with two relaxation times.

Abd-Alla et al. [20] concerned with the investigation of the reflection of generalized magneto-thermoviscoelastic waves at the boundary of a semi-infinite solid subjected to a constant temperature and magnetic field. Othman [21] has studied generalized thermo-viscoelasticity in case of a two-dimensional thermal shock problem for a finite conducting half-space. Abd-Alla and Abo-Dahab [22] made an attempt to estimate the influence due to a time harmonic normal point load or thermal source in a homogeneous isotropic magneto-thermo-viscoelastic half space using Hankel transform. Recently, the thermo-viscoelastic interaction in a homogeneous, infinite Kelvin-Voigt type viscoelastic, thermally conducting medium due to the presence of periodically varying heat sources has been studied by Kanoria and Mallik [23].

The objective of the present investigation is to determine the components of displacement, stress, temperature distribution and concentration in an isotropic homogeneous viscoelastic solid with generalized electro-magneto thermoelastic diffusion subjected to normal and thermal loads based on the G-L theory. Normal mode analysis is used to obtain the exact solutions for the variables considered. The analysis presented here involves magnetic, diffusion and viscosity parameters simultaneously which is lacking in the published literature.

2. Formulation of the problem and basic equations

We assume an isotropic, homogeneous, linear, thermally and electrically conducting viscoelastic half-space ($z \geq 0$) with diffusion and z-axis pointing vertically inwards as shown in Fig. A. The surface ($z=0$) of the half-space is taken to be traction free and subjected to mechanical and thermal loads. All considered functions are assumed to be bounded and vanish as $z \to \infty$. The whole body is at a constant temperature T_0. We consider that the orientation of the primary magnetic field $\overrightarrow{H}=(0,H_0,0)$ is towards the positive direction of y-axis. Due to the application of this magnetic field, there arises in the medium an induced magnetic field $\overrightarrow{h}$ and an induced electric field $\overrightarrow{E}$. We assume that both $\overrightarrow{h}$ and $\overrightarrow{E}$ are small in magnitude in accordance with the assumptions of the linear theory of thermoelasticity. All the considered functions will depend on time t and the coordinates x and z. So the displacement vector $\overrightarrow{u}$ will have the components

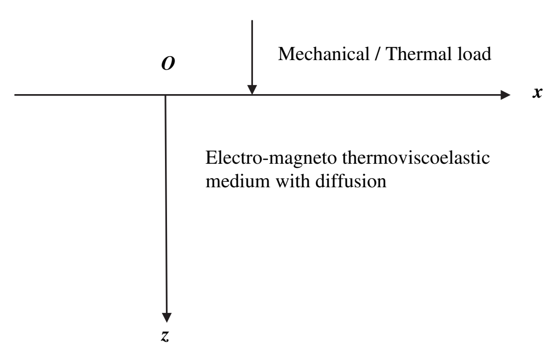

Fig. A. Geometry of the problem.