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Characteristics of mixed convection heat transfer in a lid-driven square cavity with various Richardson and Prandtl numbers

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A R T I C L E I N F O

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A B S T R A C T

In mixed convection flows, a common knowledge is that the heat transfer in a cavity is increased with increasing Grashof or Reynolds number when its respective Reynolds or Grashof number is kept at constant. On the other words, the heat transfer would increase if the flow proceeds toward pure natural convection or forced convection dominated regimes. An unanswered question is that would the heat transfer be increased continuously with simultaneously increasing both Grashof and Reynolds numbers, while keeping the Richardson and Prandtl numbers constant. And to what extent the mixed convection flows would change from laminar to chaos. These questions motivate the present study to systematically investigate the flow and heat transfer in a 2-D square cavity where the flow is induced by a shear force resulting from the motion of the upper lid combined with buoyancy force due to bottom heating. The numerical simulations cover a wide range of Reynolds ($10 \le Re \le 2200$), Grashof ($100 \le Gr \le 4.84 \times 10^6$), Prandtl ($0.01 \le Pr \le 50$), and Richardson ($0.01 \le Ri \le 100$) numbers. The average Nusselt numbers are reported to illustrate the influence of flow parameter variations on heat transfer, and they are also compared with the reported Nusselt number correlations to validate the applicability of these correlations in laminar flow regimes. Time traces of the total kinetic energy and average Nusselt number are presented to demonstrate the transition of the flows from laminar to chaos.

1. Introduction

Mixed convection heat transfer is perhaps one of the most frequently encountered physical processes in applied engineering, such as solar collectors, cooling of electronic devices, heat exchangers, materials processing, crystal growth, float glass production, metal coating and casting, and among others. In order to understand the complex physical phenomena associated with fluid flow and heat transfer, numerous studies of mixed convection driven by a combination of buoyancy and shear forces in rectangular or square cavities have been reported extensively in the literature. The study of such a problem is generally grouped into the horizontal [1–10] or vertical [11–15] side wall sliding lid-driven cavity problems except that the horizontal or vertical walls are differentially heated. In most of these studies, the authors were mainly concentrated on identifying the flow regimes and heat transfer characteristics when the Richardson number ($Ri = Gr/Re^2$) was varied, viz. the flow and heat transfer is dominated by forced convection when $Ri \leqq 1$, it is dominated by natural convection as $Ri \geqq 1$, and it is a mixed regime when Ri is of the order of 1.

In mixed convection flows, the parameters that affect heat transfer are Pr, Gr, and Re. The Gr and Re are generally grouped into a single parameter Ri. The variation of Ri is made by either changing Gr or Re and keeping one of these two parameters fixed. It has been shown that the heat transfer, in terms of Nusselt number, in a lid-driven cavity increases with increasing Pr, if Gr and Re are kept constant [2]. For a fixed Pr, the heat transfer is increased with increasing Gr or Re when its respective Re or Gr is kept constant [2,5,7,9,10,15]. An unanswered question is that would the heat transfer be increased continuously with simultaneously increasing both Gr and Re, while keeping Ri and Pr constant. And to what extent the mixed convection flows would change from laminar to chaos. These questions have not been addressed in the literature. Moreover, the characterization of heat transfer performance in mixed convection flows is generally made by evaluating the magnitude of Nusselt number. In order to provide useful information for design applications, the experimentally measured mean heat flux values [7] and numerically calculated average Nusselt number [2,16] were used to produce Nusselt number correlations. These Nusselt number correlations are assumed to be valid for a wide range of mixed convection

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Nomenclature

g	gravitational acceleration
Gr	Grashof number, $Gr = g\beta H^3(T_h - T_c)/\nu^2$
H	height of the cavity
L	width of the cavity
n	normal direction
Nu	Nusselt number
Pr	Prandtl number, ν/α
Re	Reynolds number, U_0H/ν
Ri	Richardson number, Gr/Re^2
t	dimensional time
T	dimensional temperature
U_0	top wall driven velocity
u, v	velocity components in the x- and y-directions
U, V	dimensionless velocity components
x, y	horizontal and vertical coordinates
X, Y	dimensionless coordinates

Greek symbols

α	thermal diffusivity
β	volumetric coefficient of thermal expansion
ν	fluid kinematic viscosity
θ	dimensionless temperature, $(T - T_c)/(T_h - T_c)$
τ	dimensionless time
ω	dimensional vorticity
Ω	dimensionless vorticity, $\omega H/U_0$
ψ	dimensional streamfunction
Ψ	dimensionless streamfunction, ψ/U_0H

Subscripts

c	cold temperature
h	hot temperature

problems. However, no validation of these correlations has been reported.

The objective of this study is to examine systematically the characteristics of flow and heat transfer in a lid-driven cavity when both Gr and Re are increased simultaneously from the steady, laminar flow regimes to a condition where Hopf bifurcation occurs. Validation of the reported Nusselt number correlations is also made in the present study to examine their applicability for mixed convection flows.

2. Model description and governing equations

The model configuration and boundary conditions are shown in Fig. 1 which is a square cavity with that the top wall is moving rightwards. The top and bottom walls are maintained isothermally at temperatures T_c and T_h, respectively, with $T_h > T_c$. The vertical side walls are thermally insulated. This configuration creates a gravitationally-unstable temperature gradient and results in natural convection even when the top wall is stationary. This case is similar to that studied by Moallemi and Jang [2] and Mohamad and Viskanta [3,4].

The square cavity has an aspect ratio of unity and the working fluid in the cavity is varied with $Pr = 0.01$, 0.71, 6, and 50. In addition, the working fluid is assumed to be incompressible and its properties are constant except in the body force term where the density varies with temperature. With the Boussinesq approximation, the 2-D governing equations, i.e., the momentum, energy, and mass conservation equations are transformed into streamfunction–vorticity (Ψ–Ω) and temperature non-dimensional forms:

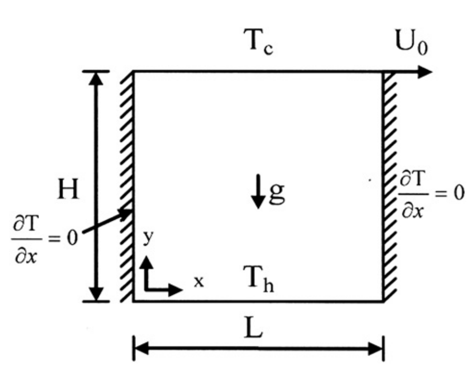

Fig. 1. Schematic diagram of computational model.

$$\frac{\partial^2\Psi}{\partial X^2} + \frac{\partial^2\Psi}{\partial Y^2} = -\Omega \quad (1)$$

$$\frac{\partial\Omega}{\partial\tau} + \left(U\frac{\partial\Omega}{\partial X} + V\frac{\partial\Omega}{\partial Y}\right) = \frac{1}{Re}\left(\frac{\partial^2\Omega}{\partial X^2} + \frac{\partial^2\Omega}{\partial Y^2}\right) + Ri\frac{\partial\theta}{\partial X} \quad (2)$$

$$\frac{\partial\theta}{\partial\tau} + \left(U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y}\right) = \frac{1}{Pr\,Re}\left(\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}\right) \quad (3)$$

$$U = \frac{\partial\Psi}{\partial Y}, \quad V = -\frac{\partial\Psi}{\partial X} \quad (4)$$

Here, the non-dimensional variables are: Ψ the streamfunction, Ω the vorticity, θ the temperature, U the X-component velocity, V the Y-component velocity, and τ the time. Ri is the Richardson number ($Ri = Gr/Re^2$), Gr is the Grashof number ($Gr = g\beta H^3\Delta T/\nu^2$), Re is the Reynolds number ($Re = U_0H/\nu$), and Pr is the Prandtl number ($Pr = \nu/\alpha$). It is noted that in the computation the original elliptic streamfunction equation is modified to become parabolic form as Eqs. (2) and (3) by inserting a false transient term to Eq. (1). If a steady state exists, the transient term in the modified parabolic form of Eq. (1) will approach to zero as is the original elliptic streamfunction equation. Each equation can be advanced through time by a direct method and the complete solution procedure may be regarded as a single iterative scheme. The dimensionless variables in the above equations are defined as

$$X = \frac{x}{H},\ Y = \frac{y}{H},\ U = \frac{u}{U_0},\ V = \frac{v}{U_0},\ \tau = \frac{tU_0}{H},$$
$$\theta = \frac{(T - T_c)}{(T_h - T_c)},\ \Omega = \frac{\omega H}{U_0},\ \Psi = \frac{\psi}{HU_0} \quad (5)$$

with u and v being the dimensional velocity components along the x and y axes, ω is the dimensional vorticity, ψ is the dimensional streamfunction, and subscripts h and c indicate the heated and cold walls, respectively. The heat transfer characteristics are described by the Nusselt number which is defined as follows:

The local Nusselt number along the isothermal walls

$$Nu(X) = -\frac{\partial\theta}{\partial Y}\bigg|_{Y=0,1} \quad (6)$$

The average Nusselt number along the isothermal walls