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A shortcut to inverse Fourier transforms: Approximate reconstruction of transient heating curves from sparse frequency domain data

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ABSTRACT

Frequency domain (AC) analysis, and associated phasor notation, offers a powerful and systematical way for dynamic thermal characterisation. The complex thermal impedance $Z_{th}(j\omega)$ plays a central role and can be obtained from analytical calculation, numerical simulation and experimental measurements. Relevant associated time domain information, such as the transient heating curve, can be derived through inverse Fourier transform (IFT). However, IFT is known to suffer from aliasing, instabilities and other artifacts. In this work we propose an alternative method that bypasses the IFT but still allows approximate reconstruction of the heating curve based on the impedance spectrum. The technique is particularly useful in cases where only truncated or sparse (low-resolution) AC data is available. It simply consists of plotting the magnitude of the impedance $|Z_{th}(j\omega)|$ (or transfer impedance for locations outside of the active junction) versus ω^{-1} as time scale. Very reasonable results, with relative errors in the order of 10%, are achieved, while the transformation is extremely simple to perform. We develop a mathematical proof for increasingly complex situations, ranging from the simple case of one single thermal time constant to a generic thermal system characterised by an arbitrary continuous time constant spectrum. Additional illustration and validation of the method is provided by practical case studies. Finally, we develop an extension to the evaluation of the impulse response and related transients. In that context the proposed method produces accurate results as well, and outperforms IFT related techniques.

1. Introduction

Frequency domain, or so called 'AC' analysis is used for dynamic characterisation in a wide range of engineering and technology domains. It consists of investigating sinusoidal oscillations of the relevant physical quantities, typically by means of a complex phasor notation. The technique is very commonly applied to electromagnetic field propagation, as encountered in e.g. transmission lines, telecommunication systems, etc. In the main context of this paper, i.e. dynamic thermal diffusion in microelectronic devices, AC analysis offers useful features as well. The use of phasors (or related variables, e.g. in the Laplace and Hankel domains) reduces the heat equation from a partial differential equation (PDE) to an ordinary differential equation (ODE), enabling easier analytical modelling and numerical simulation. Employing such transformations, Hui has developed a transmission line model for heat conduction in multilayer thin films [1]. For numerical simulations, various boundary element method (BEM) based thermal solvers, directly operating in the frequency domain, are available in the literature [2,3]. These schemes avoid the risk of unstable solutions associated with 'time marching' techniques in the time domain. As far as experimental analysis is concerned, frequency domain measurements are known to be more robust to noise than their time domain counterparts [4]. Accurate, high-resolution thermal imaging of electronic ICs can be achieved relatively easily by means of thermoreflectance, associated to a heterodyne lock-in technique [5,6]. A major advantage is that both amplitude and phase distributions are obtained. The latter can be used as a sensitive and reliable heat detector for regions where only minor temperature rises occur, which are hardly resolved by magnitude or time domain recordings. In addition, AC techniques allow to study Joule and Peltier heating/cooling modes separately, which is very useful for the characterisation and optimisation of thermoelectric and thermionic microcoolers. The Joule and Peltier effects respectively have a quadratic and linear dependence on the supplied current, and will therefore manifest themselves at different harmonics when a sinusoidal excitation with zero DC offset is used.

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Nomenclature

Roman

$a(t)$	normalised heating curve [K/W or −]
$a^*(t)$	approximated normalised heating curve [K/W or −]; absolute error: $a(t) - a^*(t)$
BEM	boundary element method
C_i	thermal capacitance in ladder network [J/K]
C_v	thermal capacitance per volume unit [J/m^3 K]
C_p	thermal capacitance per mass unit [J/kg K]
d	substrate thickness
$E(j\omega)$	excitation
f	frequency [Hz]
$F(u, v)$	generic integral kernel for $a^2(z)$
$F'(u, v)$	$\frac{dF}{dz}$
$g(z)$	modified impulse response $\frac{da}{dz}$
$g^*(z)$	approximated modified impulse response $\frac{da^*}{dz}$
$G(\vec{r}\mid\vec{r}')$	3-D Green's function [K/W]
$G(u, v)$	generic integral kernel for $a^{*2}(z)$
$G'(u, v)$	$\frac{dG}{dz}$
$H(j\omega)$	transfer function
$h(t)$	impulse response $\frac{da}{dt}$
$h^*(t)$	approximated impulse response $\frac{da^*}{dt}$
IFT	inverse Fourier transform
j	imaginary unit
k	thermal conductivity [W/m K]
P	power [W]
R_{th}	thermal resistance [K/W]
R_i	thermal resistance in ladder network [K/W]
$R(t)$	transient response
$R(\zeta)$	time constant spectrum [K/J]
R	radius of circular heat source [m]
$\vec{r}$	place vector $x\vec{1}_x + y\vec{1}_y + z\vec{1}_z$;
	relative error: $[a(t) - a^*(t)]/a(t)$
S	cross-section area [m^2]
t	time [s]
T	temperature [K]
TIM	thermal interface material
$U(j\omega)$	Fourier transform of Heaviside step function
u	transformed integration variable $u = \zeta_1 - z$
v	transformed integration variable $v = \zeta_2 - z$
$Z_{th}(j\omega)$	thermal impedance [K/W]
z	logarithmic time variable $z = \ln(t)$

subscripts

0	characteristic value
i	node or element number in ladder network
s	source

superscripts

$*$	approximated
trans	transfer

Greek

α	magnitude ratio for 2 time constant case
β	time constant ratio for 2 time constant case
γ	parameter in Fuoss–Kirkwood time constant distribution
δ	Dirac distribution
ϕ	angle variable in cilindrical coordinates (r, ϕ, z)
ρ	mass density [kg/m^3]
τ	thermal time constant [s]
ω	angular frequency $\omega = 2\pi f$ [rad/s]
ξ	dimensionless time variable [−]
ζ	logarithmic time constant variable $\zeta = \ln(\tau)$

Despite the inherent advantages offered by frequency domain techniques, and the fact that AC analysis in itself provides a complete, systematic dynamic characterisation of the system, additional transient information is desirable in many situations. The normalised heating curve $a(t)$, i.e. the temperature response to a 1 W power step, is of particular interest. The reason is that through adequate further processing, useful dynamic information such as the thermal time constant spectrum and structure functions can be derived [7–9]. In principle, the transient response $R(t)$ to an arbitrary excitation $E(j\omega)$ can be evaluated by inverse Fourier transform (IFT) if the transfer function $H(j\omega)$ of the system is known:

$$R(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)E(j\omega)\exp(j\omega t)\mathrm{d}\omega \qquad (1)$$

Unfortunately, the IFT is known to suffer from aliasing effects and other artifacts. These issues become especially problematic when only low-resolution samples or data truncated to a finite frequency interval for H is available, as is typically encountered when dealing with experimental measurements. Additional difficulties to evaluate the integral arise when the transfer function has poles on the imaginary axis.

These problems associated with the IFT are not new, and have been addressed by several authors. Godinho et al. [2] return to the time domain by inserting complex frequencies with a small imaginary part, of the form $\omega_c = \omega - j\eta$, into the IFT. The extra phase shift introduced is then accounted for by applying an exponential window of the form $\exp(\eta t)$ to the obtained transient. Other authors apply a window function to the frequency spectrum instead to compensate truncation errors [10], and proposed similar modified IFTs to tackle imaginary poles [11]. The application of such window functions may however add false oscillations to the transient, and blur the initial parts. Krylov and Liakishev [12] developed a projection technique for Fourier inversion of data truncated to a finite interval, that avoids window functions and is based on the expansion of the frequency spectrum into Hermite eigenfunctions. Transient analysis of electromagnetic field propagation by IFT has received further particular attention. Rachidi et al. [13] performed a low-frequency series expansion of the ground impedance matrix of multiconductor lines above a lossy ground such that the Fourier inversion of the matrix elements could be carried out semi-analytically, with careful treatment of the singularities. They used this technique to investigate lightning-induced voltages in the lines. Shi [14] approximates a bounded, causal transfer function $H(j\omega)$ as a sum of complex exponentials. The transient response of each of the terms can then be obtained through exact inverse Fourier transform. However, the decomposition of H involves computationally costly matrix operations including singular value decomposition and QR factorisation into an orthogonal and right triangular matrix.

In this paper we propose a method, mainly targeted at heat transfer applications, to evaluate transient responses from AC information but entirely bypassing inverse Fourier transforms. The technique is applicable to sparse frequency spectrum data and very