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Optimization of fractal-like branching microchannel heat sinks for single-phase flows

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ABSTRACT

Fractal-like branching flow networks in disk-shaped heat sinks are numerically optimized to minimize pressure drop and flow power. Optimization was performed using a direct numerical search, gradient-based optimization, and genetic algorithm. A previously validated one-dimensional pressure drop and heat transfer model, with water as the working fluid, is employed as the objective function. Geometric constraints based on fabrication limitations are considered, and the optimization methodology is compared with results from a direct numerical search and a genetic algorithm.

The geometric parameters that define an optimal flow network include the length scale ratio, width scale ratio, and terminal channel width. Along with disk radius, these parameters influence the number of branch levels and number of channels attached to the inlet plenum. The geometric characteristics of the optimized flow networks are studied as a function of disk radius, applied heat flux, and maximum allowable wall temperature. A maximum inlet plenum radius, minimum interior channel spacing, and ranges of terminal channel widths and periphery channel spacing are specified geometric constraints. In general, all geometric constraints and the heat flux have a significant influence on the design of an optimal flow network. Results from a purely geometrically derived network design are shown to perform within 15% of the direct search and gradient-based optimized configurations.

1. Introduction

Disk-shaped heat sinks with branching microchannels were first proposed by Pence [1] as a means to reduce both the pressure drop and the maximum streamwise wall temperature difference observed in parallel microchannel heat sinks. A fractal-like branching flow network, inspired by nature, was studied. Representative fractal-like flow networks in a disk-shaped heat sink configuration are shown in Fig. 1. Flow enters the network from the inlet plenum located at the center of the disk and exits at the periphery. Each channel emanating from the inlet plenum bifurcates into two narrower channels, each of which in turn bifurcates. This repetitive pattern is considered fractal-like because the ratio of the channel widths and channel lengths between the consecutive branch levels are fixed. The objective of the present study is to develop an optimization algorithm to identify geometric characteristics, subject to operating and fabrication constraints, of a *fractal-like* flow network that achieves the minimum single-phase flow power for a desired heat removal.

The gradient-based optimization algorithm used in the present study employs the one-dimensional model developed by Pence [2] for predicting pressure and wall temperature distributions in fractal-like branching channel networks. The model is based on developing laminar flow and heat transfer under a constant wall heat flux condition at all four walls. Hydrodynamic [3] and thermal [4] boundary layers were assumed to redevelop following each bifurcation. Alharbi et al. [5,6] validated the model using three-dimensional computational fluid dynamic and thermal analyses, respectively. Using the one-dimensional model of Pence [2], Pence and Enfield [7] investigated the influence of several geometric parameters on the pressure drop and maximum wall temperature. It was determined that increasing the number of branch levels resulted in reduced flow resistance, but also resulted in an increase in the maximum predicted wall temperature.

Using *constructal theory*, Bejan [8] designed optimal flow networks that minimized global flow resistance between a single point and a volume. Bejan and Errera [9] extended the analysis to simultaneously minimize flow and thermal resistance while optimizing the cooling of a volume experiencing uniform heat generation. Using the disk-shaped concept proposed by Pence [1], Lorente et al. [10] employed constructal theory to minimize cooling path lengths from the center of a planar disk to a series of equally spaced

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Nomenclature

A	surface area (m^2)
a	ratio of maximum plenum radius to disk radius (r/R)
b	terminal channel spacing parameter
c_p	specific heat capacity of fluid (J kg^{-1} K^{-1})
d	internal channel spacing parameter
H	height/depth (m)
k	local branch level (from 0 to m)
L	length (m)
m	total number of branch levels k
n_k	number of channels per branch level k
ΔP	pressure drop (Pa)
$\wp$	flow power $\dot{V}_{tot}\Delta P$ (W)
q''	heat flux applied to heat sink surface (W/m^2)
r	inlet plenum radius (m)
R	disk radius (m)
T	temperature (°C)
$V^{\dot{Y}}$	volumetric flow rate (l/s)
w	channel width (m)
w_m^*	non-dimensional channel width in level m, w_m/H

Greek symbols

β	width scale ratio, Eq. (1)
ε_p	benefit-to-cost ratio, Eq. (9)
γ	length scale ratio, Eq. (2)
ρ	fluid density (kg/m^3)

Subscripts

0	$k = 0$ level branch
ex	exit
in	inlet
k	branch level index
m	terminal branch level
max	maximum
min	minimum
tot	total
w	channel wall

points on the disk circumference. It was concluded that minimizing channel lengths yielded flow resistances very similar to those based on a minimization of flow resistance, which were presented in a parallel study by Wechsatol et al. [11]. In the same disk configuration, Wechsatol et al. [12] later minimized both the flow and thermal resistances. For a fixed flow rate the thermal resistance was minimized with non-branching channels, whereas flow resistance was minimized with increased numbers of branch levels. In these optimization studies of constructal networks [8–12], the flow was assumed laminar with fully developed flow, variable length and width scale ratios were considered, and a fixed coolant volume was imposed. Gonzales et al. [13], also assuming fully developed flow, optimized the heat transfer and fluid resistances by varying the number of downstream branches coupled to an upstream branch.

In the present optimization study of fractal-like flow networks, more restrictive fabrication constraints than have been previously considered are taken into account. In addition to a maximum wall temperature constraint, the present study also includes a restriction on the inlet plenum diameter, which is highly influenced by the number of channels connected to the inlet plenum. Several constraints imposed by the fabrication process, such as the minimum channel width and minimum spacing between channels to allow sufficient bonding area, are also imposed. Optimization is performed for a fixed physical space, in this case the disk diameter, as opposed to a fixed fluid volume. The objective of the optimization is to find the flow network that minimizes flow power or pressure drop while providing a required cooling load and adhering to a maximum wall temperature constraint. Results of the gradient-based optimization are validated with a direct search over the entire range of variables and are compared with results from a genetic algorithm.

In the direct numeric search, each parameter is varied at discrete intervals over a specified range. The flow power for each parametric permutation is computed using the one-dimensional model. Finally a search of the entire set of results is conducted to determine the geometry that minimizes flow power, the process being very time intensive.

The gradient-based steepest descent search starts at a single point within the parameter space, evaluates the gradient, and moves in incremental steps in the direction of the minimum [14]. The steepest descent approach has two distinct advantages over the direct search. First, there is a reduction in the number of executions of the one-dimensional model to determine the optimized flow network. The second advantage is continuous parameters with finer resolution than can be achieved using the direct search and its discrete intervals.

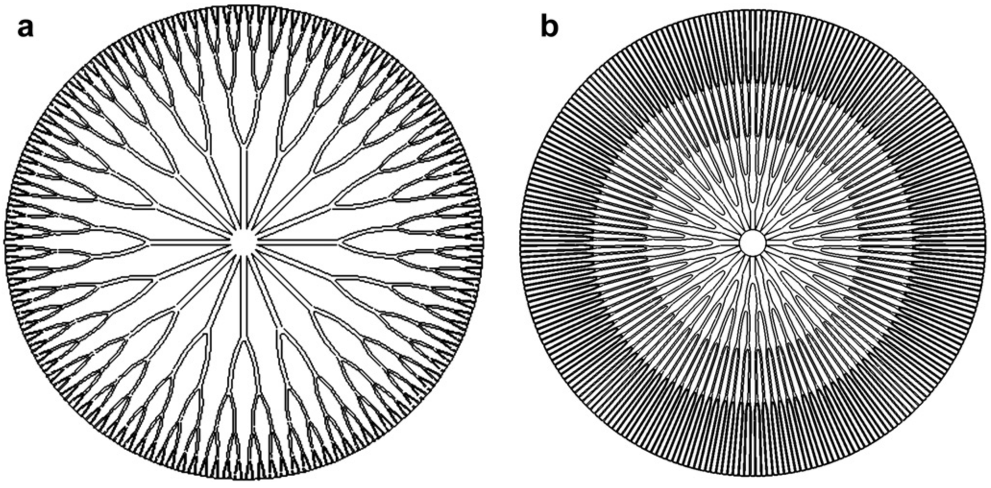

Fig. 1. Representative flow networks with 16 branches emanating from the inlet plenum at the center of the disk (a) decreasing channel length, and (b) increasing channel length.