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Exact solution for size-dependent elastic response in laminated beams considering generalized first strain gradient elasticity

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A B S T R A C T

In this paper, exact solutions for the static bending response of laminated composite beams have been derived considering the effect of strain gradient elasticity. Separate solutions for the laminate comprising of isotropic and orthotropic laminae have been presented here. The sixth-order tensor for the higher order elastic coefficients has been evaluated using the generalized first strain gradient elasticity model with three independent material length constants. The significance of the gradient elasticity in low-dimensional structures has been established employing numerical examples of micro and nano-beams, and comparing the current results with the classical elasticity results. A parametric study over the variation of the axial, transverse Cauchy and physical stresses across the thickness of the beam has been conducted. Also, the discontinuity of the inter-laminar transverse Cauchy stresses and higher order stresses across the thickness has been studied in the current framework. The exact solutions developed in this paper may be used as benchmark results for validating further research involving the strain gradient elastic response of low-dimensional laminated composite beams.

1. Introduction

Micro- and nano-electro mechanical systems are being extensively used as micro-sensors and actuators [1–4]. However, the elastic behaviour of these structures when measured experimentally have shown significant deviation from the classical elastic results [5–7]. Considering the use of these devices in sensing and actuation of sensitive applications, this difference is unacceptable. The micro-structural effects at these low dimensions are caused by the non-local interaction of the stress and strain in the system [8]. This non-local phenomenon resulting in a size-dependent behaviour causes a significant effect over the mechanical response of the low-dimensional structures.

Mindlin [9,10] proposed the first and second strain gradient theories for linear elastic continuum, which can be used to account for this behaviour. These gradient models consider the material length constants for evaluation of the structural response. Owing to the difficulty in calculating 16 material length constants necessary for an isotropic material, these models have been simplified. Mindlin and Eshel [11] proposed the first strain gradient elasticity model involving only five independent material length constants for isotropic bodies. Parallely, Toupin [12] and Mindlin [13] also modelled the gradient effects using the couple stress theory involving two independent material length constants. Yang et al. [14] modified this couple stress model to derive the modified couple stress theory involving only a single material length constant considering only the symmetric component of the couple stress. Hadjesfandiari [15] presented a model considering only the anti-symmetric component of the couple stress theory. Lam et al. [5] extended the Yang's couple stress model [14] to derive the strain gradient theory involving the symmetric component of couple stress along with the dilatational and stretch gradients. Experimental studies to determine the material length scales based on these theories have been conducted [16–19]. All these models consider the first gradients of strain in a simplified form. The physical inconsistencies owing to the assumptions over the symmetric couple stress have been discussed, and the asymmetric couple stress has been mathematically established by Shaat [20]. Zhou et al. [21] reformulated the Mindlin's first strain gradient elasticity model [11] to derive a strain gradient model for isotropic materials involving three independent material length constants. This model is a more generalized form of the Lam's model considering both the symmetric and anti-symmetric components of curvatures. This model may be considered as the Generalized First Strain Gradient Theory (GFSGT).

Numerous studies involving strain gradient effects in structures have been carried out. The static and dynamic response of beams considering Euler–Bernoulli theory [22–24], Timoshenko theory [25,26], classical laminated plate theory [27,28], Mindlin plate theory [29,30] have been evaluated. All the above studies were for homogeneous isotropic materials. Further studies were extended to functionally graded beams [31] and laminated composites [32–35]. However, the above studies were based on simplified models of strain gradient elasticity, and have assumed simpler beam theories trivially resulting in zero strain gradient components. The authors have derived

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the exact solutions for the static bending response of a simply supported homogeneous isotropic beam based on GFSGT, by solving the governing equations and associated boundary conditions [36]. By solving for the displacements from the governing equations, no assumptions were made over the displacement fields in their analysis. Later, they have extended their study to derive the numerical solution for other general loading and boundary conditions [37] by considering the higher order deformation theory. Realizing the importance of laminated composites for structural applications, a similar analysis is deemed necessary for determining the size-dependent bending response of the laminated beams.

Therefore, this paper deals with the derivation of the exact solutions for the size-dependent elastic response of simply-supported laminated beams subjected to a sinusoidally distributed mechanical load. The size-effects over the elastic response have been modelled by the GFSGT. The governing equations and the boundary, interface conditions necessary for this have been derived from the variational principles. These exact solutions may be used as benchmark for validating further research involving micro- and nano-laminated composites.

2. Constitutive relations

The deformation energy density of an elastic continuum considering the strain gradients may be written as [36]:

$$W(\epsilon_{ij}, \eta_{ijk}) = \frac{1}{2}C_{ijkl}\,\epsilon_{ij}\epsilon_{kl} + \frac{1}{2}g_{ijklmn}\eta_{ijk}\eta_{lmn} \tag{1}$$

here, the strain tensor, ϵ_{ij} and the strain gradient tensor, η_{ijk} are expressed in terms of the displacement vector u_i as:

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \eta_{ijk} = \epsilon_{jk,i} \tag{2}$$

The size-effects being analyzed in the current study are due to the inclusion of the energy corresponding to the strain gradients in Eq. (1) [36,21]. Based on the above form of the deformation energy, the second-order Cauchy stress tensor $\boldsymbol{\sigma}$, and the third-order moment stress tensor $\boldsymbol{\tau}$, which are the energy conjugates of the strain and the strain gradient tensor respectively, may be expressed as:

$$\sigma_{ij} = C_{ijkl}\,\epsilon_{kl}, \quad \tau_{ijk} = g_{ijklmn}\eta_{lmn}, \tag{3}$$

Based on the above definitions for the stress and the higher order stress tensors, the deformation energy density given by Eq. (1) may be rewritten as:

$$W(\epsilon_{ij}, \eta_{ijk}) = \frac{1}{2}\sigma_{ij}\,\epsilon_{ij} + \frac{1}{2}\tau_{ijk}\eta_{ijk} \tag{4}$$

In the above constitutive relations, the fourth order tensor C_{ijkl} denotes the conventional fourth order elastic coefficient tensor, and g_{ijklmn} is the sixth order higher order elasticity tensor. The forms of the fourth-order elastic tensor for all forms of material symmetry are well documented [38]. The higher order elasticity tensor $\mathbf{g}$ for an isotropic solid has been proposed as [21]:

$$\begin{aligned} g_{ijklmn} = {} & \tfrac{a_1}{4}(\delta_{ij}\delta_{kl}\delta_{mn} + \delta_{im}\delta_{jk}\delta_{ln} + \delta_{ik}\delta_{jl}\delta_{mn} + \delta_{in}\delta_{jk}\delta_{lm}) \\ & + \tfrac{a_2}{4}(\delta_{ij}\delta_{km}\delta_{ln} + \delta_{ij}\delta_{kn}\delta_{lm} + \delta_{ik}\delta_{jm}\delta_{ln} + \delta_{ik}\delta_{jn}\delta_{lm}) \\ & + a_3\delta_{il}\delta_{jk}\delta_{mn} + \tfrac{a_4}{2}(\delta_{il}\delta_{jm}\delta_{kn} + \delta_{il}\delta_{jn}\delta_{km}) \\ & + \tfrac{a_5}{4}(\delta_{im}\delta_{jl}\delta_{kn} + \delta_{im}\delta_{jn}\delta_{kl} + \delta_{in}\delta_{jl}\delta_{km} + \delta_{in}\delta_{jm}\delta_{kl}) \end{aligned} \tag{5}$$

where, the material constants $a_i (i = 1 \cdots 5)$ are given in terms of the Lamé parameter μ, and the three micro-structure dependent material scales l_0, l_1 and l_2 as:

$$\begin{aligned} & a_2 = \tfrac{2}{15}(27\mu l_0^2 - 4\mu l_1^2 - 15\mu l_2^2), \quad a_4 = \tfrac{2}{3}(\mu l_1^2 + 6\mu l_2^2), \quad a_5 = \tfrac{4}{3}(\mu l_1^2 - 3\mu l_2^2), \\ & a_1 = -\tfrac{2}{3}(a_2 + a_5), \quad a_3 = \tfrac{2}{3}a_2 + \tfrac{1}{6}a_5 \end{aligned} \tag{6}$$

3. Strain gradient elasticity governing equations

For an elastic solid of volume Ω, bounded by $\partial\Omega$, with sharp edges Γ, the total deformation energy of the solid may be written as:

$$U = \int_\Omega W(\epsilon_{ij}, \eta_{ijk})dV \tag{7}$$

The corresponding governing differential equations of equilibrium, and the associated boundary conditions may be obtained from applying the variational principle over U. Using Eq. (4) in Eq. (7), the first variation of the total deformation energy of the solid can be written as:

$$\delta U = \int_\Omega \sigma_{ik}\delta u_{k,i} + \tau_{ijk}\delta u_{k,ji}dV \tag{8}$$

This may be expressed in terms of the mutually independent variations of the displacement vector δu_i, and the normal gradient of the displacement vector $D\delta u_i = n_k(\partial\delta u_i/\partial x_k)$ as:

$$\begin{aligned} \delta U = {} & \int_\Omega (-\sigma_{ik,i} + \tau_{ijk,ij})\delta u_k dV + \int_{\delta\Omega} (n_i(\sigma_{ik} - \tau_{ijk,j}) + (D_l n_l)n_j n_i \tau_{ijk} \\ & - D_i(n_j\tau_{ijk}))\delta u_k dA + \int_{\partial\Omega} n_i n_j \tau_{ijk} D(\delta u_k)dA + \oint_\Gamma [[m_i n_j \tau_{ijk}]]\delta u_k dl \end{aligned} \tag{9}$$

where, $D_j(\cdot) = (\delta_{jl} - n_j n_l)\partial(\cdot)/\partial x_l$ denotes the surface gradient.

A detailed derivation of the above equation has been carried out by the authors in their previous article [36], and hence not repeated here for brevity. Considering the variation of the external work done (V) over the solid we may write:

$$\delta V = \int_\Omega \bar{b}_k \delta u_k dV + \int_{\partial\Omega} (\bar{t}_k\delta u_k + \bar{q}_k D\delta u_k)dA + \oint_\Gamma \bar{r}_k \delta u_k dl \tag{10}$$

where, $\bar{b}_k$ is the applied body force density, $\bar{t}_k$, $\bar{q}_k$ are the surface traction and double stress traction vector applied per unit surface area, respectively, and $\bar{r}_k$ is line load over unit length along sharp edge. Using the variational principle $\delta U = \delta V$ the following governing differential equations are derived from Eqs. (9) and (10):

$$\sigma_{ik,i} - \tau_{ijk,ij} + \bar{b}_k = 0, \quad \text{in}\,\Omega \tag{11}$$

The variational principle also yields the following associated boundary conditions:

$$(n_i(\sigma_{ik} - \tau_{ijk,j}) + (D_l n_l)n_j n_i \tau_{ijk} - D_i(n_j\tau_{ijk})) = \bar{t}_k \text{ or } u_k = \overline{u_k} \text{ on } \partial\Omega \tag{12}$$

$$n_i n_j \tau_{ijk} = \bar{q}_k \text{ or } Du_k = \overline{Du_k} \text{ on } \partial\Omega \tag{13}$$

$$[[m_i n_j \tau_{ijk}]] = \bar{r}_k \text{ or } u_k = \overline{u_k} \text{ on } \Gamma \tag{14}$$

4. Exact solutions for lamina

Fig. 1 illustrates a simply supported laminated beam in the x_1x_3-plane. The geometric length, and the width of the laminate are L and b, respectively. The laminated beam is comprised of N laminae, and the total geometric height of the beam is h. The Cartesian coordinate

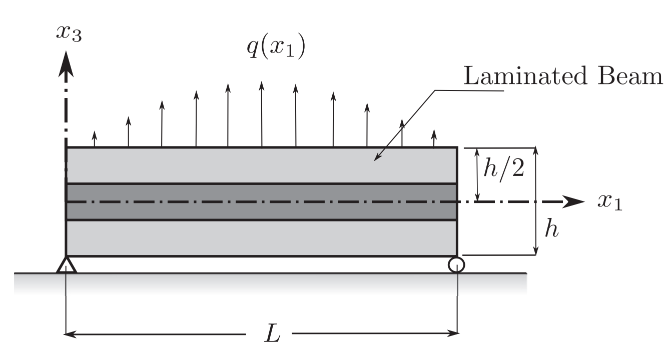

Fig. 1. A schematic of the simply-supported laminated beam with a sinusoidally distributed load applied at $x_3 = h/2$.

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