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The MLPG for crack analyses in composites with flexoelectricity effects

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ABSTRACT

The meshless Petrov-Galerkin (MLPG) method is developed to analyse 2-D crack problems where the electric field and displacement gradients exhibit a size effect. The size-effect phenomenon in micro/nano electronic structures is described by the strain- and electric field-gradients. Both the electric intensity vector and strain gradients are considered in the constitutive equations of the material and the governing equations are derived with the corresponding boundary conditions using the variational principle. The coupled governing partial differential equations (PDE) for stresses and electric displacement field are satisfied in a local weak-form on small fictitious subdomains. All field quantities are approximated by the moving least-squares (MLS) scheme. After performing the spatial integrations, we obtain a system of algebraic equations for the nodal unknowns.

1. Introduction

The size-effect phenomenon is observed in structures where characteristic length of material structure is compared with the size of the analyzed body. The size-effect has been observed in a number of experiments [1–7]. The electric intensity vector- and strain-gradient effect is very strong mainly for nano-sized dielectrics. The classical continuum mechanics neglects the interaction of material microstructure and the results are size-independent. The atomistic models have been developed to consider size-effect phenomena in materials. Unfortunately, there are extremely high requirements on computer memory in atomistic models. It seems to be more convenient to develop advanced continuum theories to account intrinsic length scales for materials. Up to date there are two reliable advanced continuum theories with size-effect inclusion, namely the non-local elasticity [8,9], and strain-gradient elasticity [10,11]. In the nonlocal theory, the stress at a reference point is a functional of strains at more points of the body and size effect parameter is considered in the weight function. In the gradient theory, there are considered also the higher order strain gradients in the strain energy density of a solid. The formulation based on two length scales [10] is very complicated and it was simplified by Aifantis [12] by introducing only one length parameter. The further mathematical and implementational treatment can be found in later works [13,14]. A review of various higher-order gradients theories has been published by Fleck and Hutchinson [15].

The large strain gradients and dislocations are occurring at the crack tip vicinity. Heterogeneous plastic deformation requires additional dislocations to ensure geometric compatibility. They contribute mainly to material work hardening. Paneda et al. [16] investigated the influence of gradient-enhanced dislocation hardening on the mechanics of notch-induced failure. It seems that the gradient elasticity theory can be suited for studying crack problems. This theory can eliminate the crack-tip strain singularity while providing structure to the cohesive zone without resorting to extraneous forces as in plastic strip models [17]. In the literature one can find a lot of applications of the gradient theory to crack problems in elasticity [18–25]. The near-tip fields for a crack in elastic or elastic–plastic materials with strain-gradient effects under mixed mode loadings are given by Huang et al. [26]. There is presently a paucity of papers devoted to fracture mechanics analysis of piezoelectric solids described by the gradient theory [27].

For more real modeling of cracks in piezoelectric solids it is needed to consider the electric field and strain gradients in a more appropriate and reliable size-dependent theory. The linear coupling of electric fields and strain gradients is called as the direct flexoelectric effect [28–30]. This phenomenon can be viewed as a higher order effect with respect to piezoelectricity. Except the direct flexoelectric effect, we have also converse flexoelectricity, where there is the linear coupling between the stress and applied electric field gradients [31–33]. Hu and Shen [34,35] have applied the variational principle for nano-sized elastic dielectrics with the flexoelectric effects as well as the surface effects to derive governing equations.

In this study, the size-effect is considered by including the strain gradients and, also, by the electric field-strain gradient coupling for in-plane crack problems in a piezoelectric body. The variational principle

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is applied to derive the governing equations for considered constitutive equations with both the direct and converse flexoelectricity. Due to the high mathematical complexity of the boundary value problem, we need accurate and efficient computational tool to solve a general problem. The C^1-continuous elements are required in numerical discretization methods to guarantee the continuity of variables and their derivatives in the present theory involving the fourth-order derivatives of displacements and electric potential in governing equations. To overcome this problem in the finite element method (FEM), it is needed to apply the mixed formulation or to develop the subparametric C^1-continuous elements [36]. However, it is a difficult task. Recently, Deng et al. [37] have developed a mixed finite element method for the study of problems with both strain gradient elasticity and flexoelectricity being taken into account. Then, the C^0 continuous elements can be used in mixed FEM and the kinematic relationship between displacement field and its gradient is enforced by Lagrangian multipliers.

It is familiar that in the Meshless Local Petrov-Galerkin method (MLPG) with the Moving Least-square (MLS) approximation the order of continuity of the MLS approximation is given by the minimum between the orders of continuity of the basis functions and that of the weight function. This allows the order of continuity to be tuned to a desired value [38–40]. However, in conventional discretization methods, such as the FEM or the boundary element method (BEM), the interpolation functions usually result in a discontinuity of secondary fields on the interfaces of elements. A conventional displacement-based FEM approach cannot be readily used to compute flexoelectricity since the C^1 continuity is required for primary fields. Abdollahi et al. [41] applied the smooth meshfree basis functions in a Galerkin method to flexoelectric problems. It allows to consider general geometries and boundary conditions.

In the present paper, the authors have developed a meshless method based on the MLPG weak-form to solve general crack problems in piezoelectric solids with both the direct and converse flexoelectricity. Numerical examples are presented and discussed to compare the results obtained by the gradient theory with those obtained by classical theory.

2. Basic equations for electric field-strain gradient theory

The gradients of electric intensity vector and strains are considered in the constitutive equations for piezoelectric solids [34,35]

$$\begin{aligned}
\sigma_{ij} &= c_{ijkl}\varepsilon_{kl} - e_{kij}E_k - b_{klij}E_{k,l},\\
\tau_{jkl} &= g_{jklmni}\eta_{mni} - f_{ijkl}E_i,\\
D_k &= a_{kl}E_l + e_{kij}\varepsilon_{ij} + f_{klmn}\eta_{lmn},\\
Q_{ij} &= h_{ijkl}E_{k,l} + b_{ijkl}\varepsilon_{kl},
\end{aligned}\tag{1}$$

where σ_{ij}, D_k, τ_{jkl} and $Q_{ij}$are the stress tensor, electric displacements, higher order stress and electric quadrupole, respectively. The material parameters **a** and **c** are the second-order permittivity and the fourth-order elastic constant tensors, respectively. The symbol **e** denotes the piezoelectric coefficient and **f** is the electric field-strain gradient coupling coefficient tensors representing the higher-order electromechanical coupling induced by the strain gradients. The sixth-order material tensor **g** is the higher-order elastic parameter. The symbols **b** and **h** denote the quadrupole-strain coefficients and higher-order electric parameters, respectively.

The strain tensor ε_{ij} and the electric field vector E_j are related to the displacements u_i and the electric potential ϕ by

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_j = -\phi_{,j} \tag{2}$$

The strain-gradient tensor $\boldsymbol{\eta}$ is defined as

$$\eta_{ijk} = \varepsilon_{ij,k} = \frac{1}{2}(u_{i,jk} + u_{j,ik}) \tag{3}$$

The two length scales in the Mindlin theory are taken equal to each other in the Aifantis theory [13]. It greatly simplifies mathematical and implementational treatment. Then, the higher-order elastic parameters, g_{jklmni}, are assumed to be proportional to the conventional elastic stiffness coefficients, c_{klmn}, by the internal length material parameter l [42,43]. Then, one can write

$$g_{jklmni} = l^2 c_{jkmn}(\delta_{l1}\delta_{i1} + \delta_{l3}\delta_{i3}) \tag{4}$$

In the gradient theory of elasticity Askes and Aifantis [14] have considered the Laplacian term in constitutive equation for stress tensor

$$\sigma_{ij} = c_{ijkl}(\varepsilon_{kl} - l^2\varepsilon_{kl,mm})$$

The electric displacement vector for an advanced theory in analogy with gradient theory of elasticity can be written as

$$D_k = a_{kl}E_l + e_{kij}[\varepsilon_{ij} + m^2(\delta_{n1} + \delta_{n3})\varepsilon_{ij,n}]$$

Similarly, the electric field-strain gradient coupling coefficients **f** and the quadrupole-strain coupling coefficients **b** are assumed to be proportional to the piezoelectric coefficients and the scaling parameter m. The higher-order electric parameters **h** can be expressed by the dielectric constants a_{kl} and the scaling parameter q. Then, one can write

$$f_{klmn} = m^2 e_{klm}(\delta_{n1} + \delta_{n3}), \quad b_{klij} = \lambda^2 e_{kij}(\delta_{l1} + \delta_{l3}), \quad h_{ijkl} = q^2 a_{ik}(\delta_{j1}\delta_{l1} + \delta_{j3}\delta_{l3}). \tag{5}$$

Making use of the standard Voigt notation for strains and stresses as well as their gradients, the constitutive Eq. (1) for 2D problems in (x_1, x_3)-plane and orthotropic materials can be written in a matrix form as

$$\begin{aligned}
\begin{bmatrix}\sigma_{11}\\ \sigma_{33}\\ \sigma_{13}\end{bmatrix} &= \begin{bmatrix} c_{11} & c_{13} & 0\\ c_{13} & c_{33} & 0\\ 0 & 0 & c_{44}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{33}\\ 2\varepsilon_{13}\end{bmatrix} - \begin{bmatrix} 0 & e_{31}\\ 0 & e_{33}\\ e_{15} & 0\end{bmatrix}\left(\begin{bmatrix}E_1\\ E_3\end{bmatrix} + \lambda^2\begin{bmatrix}E_{1,1}\\ E_{3,1}\end{bmatrix} + \lambda^2\begin{bmatrix}E_{1,3}\\ E_{3,3}\end{bmatrix}\right)\\
&= \mathbf{C}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{33}\\ 2\varepsilon_{13}\end{bmatrix} - \mathbf{L}\left(\begin{bmatrix}E_1\\ E_3\end{bmatrix} + \lambda^2\begin{bmatrix}E_{1,1}\\ E_{3,1}\end{bmatrix} + \lambda^2\begin{bmatrix}E_{1,3}\\ E_{3,3}\end{bmatrix}\right)
\end{aligned}\tag{6}$$

$$\begin{aligned}
\begin{bmatrix}D_1\\ D_3\end{bmatrix} &= \begin{bmatrix} a_{11} & 0\\ 0 & a_{33}\end{bmatrix}\begin{bmatrix}E_1\\ E_3\end{bmatrix} + \begin{bmatrix} 0 & 0 & e_{15}\\ e_{31} & e_{33} & 0\end{bmatrix}\left(\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{33}\\ 2\varepsilon_{13}\end{bmatrix} + m^2\begin{bmatrix}\varepsilon_{11,1}\\ \varepsilon_{33,1}\\ 2\varepsilon_{13,1}\end{bmatrix}\right.\\
&\quad\left. + m^2\begin{bmatrix}\varepsilon_{11,3}\\ \varepsilon_{33,3}\\ 2\varepsilon_{13,3}\end{bmatrix}\right) = \mathbf{A}\begin{bmatrix}E_1\\ E_3\end{bmatrix} + \mathbf{L}^T\left(\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{33}\\ 2\varepsilon_{13}\end{bmatrix} + m^2\begin{bmatrix}\varepsilon_{11,1}\\ \varepsilon_{33,1}\\ 2\varepsilon_{13,1}\end{bmatrix}\right.\\
&\quad\left. + m^2\begin{bmatrix}\varepsilon_{11,3}\\ \varepsilon_{33,3}\\ 2\varepsilon_{13,3}\end{bmatrix}\right)
\end{aligned}\tag{7}$$

$$\begin{aligned}
\begin{bmatrix}\tau_{11l}\\ \tau_{33l}\\ \tau_{13l}\end{bmatrix} &= l^2\begin{bmatrix} c_{11} & c_{13} & 0\\ c_{13} & c_{33} & 0\\ 0 & 0 & c_{44}\end{bmatrix}\begin{bmatrix}\varepsilon_{11,l}\\ \varepsilon_{33,l}\\ 2\varepsilon_{13,l}\end{bmatrix} - m^2(\delta_{l1} + \delta_{l3})\begin{bmatrix} 0 & e_{31}\\ 0 & e_{33}\\ e_{15} & 0\end{bmatrix}\begin{bmatrix}E_1\\ E_3\end{bmatrix}\\
&= l^2\mathbf{C}\begin{bmatrix}\varepsilon_{11,l}\\ \varepsilon_{33,l}\\ 2\varepsilon_{13,l}\end{bmatrix} - m^2(\delta_{l1} + \delta_{l3})\mathbf{L}\begin{bmatrix}E_1\\ E_3\end{bmatrix}, \quad \text{for } l = 1,\ 3
\end{aligned}\tag{8}$$

$$\begin{aligned}
\begin{bmatrix}Q_{1j}\\ Q_{3j}\end{bmatrix} &= q^2\begin{bmatrix} a_{11} & 0\\ 0 & a_{33}\end{bmatrix}\begin{bmatrix}E_{1,j}\\ E_{3,j}\end{bmatrix} + \lambda^2(\delta_{j1} + \delta_{j3})\begin{bmatrix} 0 & 0 & e_{15}\\ e_{31} & e_{33} & 0\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{33}\\ 2\varepsilon_{13}\end{bmatrix}\\
&= q^2\mathbf{A}\begin{bmatrix}E_{1,j}\\ E_{3,j}\end{bmatrix} + \lambda^2(\delta_{j1} + \delta_{j3})\mathbf{L}^T\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{33}\\ 2\varepsilon_{13}\end{bmatrix}. \quad \text{for } j = 1,\ 3
\end{aligned}\tag{9}$$

A piezoelectric solid occupies the domain V with the boundary Γ. The electric Gibbs free energy density function $\mathscr{U}$ in the electric field gradient theory is given by Hu and Shen [34] as

$$\mathscr{U} = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} + \frac{1}{2}\tau_{ijk}\eta_{ijk} + \frac{1}{2}D_i\phi_{,i} - \frac{1}{2}Q_{ij}E_{i,j} \tag{10}$$