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Full field computing for elastic pulse dispersion in inhomogeneous bars

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ABSTRACT

In the paper, the finite element method and the finite volume method are used in parallel for the simulation of a pulse propagation in periodically layered composites beyond the validity of homogenization methods. The direct numerical integration of a pulse propagation demonstrates dispersion effects and dynamic stress redistribution in physical space on the example of a one-dimensional layered bar. Results of numerical simulations are compared with the analytical solution constructed specifically for the considered problem. The analytical solution as well as numerical computations show the strong influence of the composition of constituents on the dispersion of a pulse in a heterogeneous bar and the equivalence of results obtained by two numerical methods.

1. Introduction

Wave propagation in a slender heterogeneous solid bar is the typical test problem for models of composite materials [1–4, e.g.]. The modeling is necessary because macroscopic properties of composite materials are strongly influenced by the properties of their constituents. The macroscopic properties are usually determined by a homogenization, which yields the effective stresses and strains acting on the effective material.

The basic idea of homogenization consists in a replacement of a heterogeneous solid by a homogeneous one which, from the macroscopic point of view, behaves in the same way, as do its constituents, but with different, effective, values of the appropriate material constants [5]. This idea reappeared many times in the last two centuries, as it is indicated in recent reviews [5–8]. Mathematical details of classical homogenization models can be found in [9].

Layered periodic materials represent the simplest possible pattern of composites from the theoretical point of view. Their modeling also has a rich history [10]. Constitutive models of effective properties for such materials are still under development using either ensemble averaging [11,12], or integration over unit cell [13,14], or scattering response [15]. However, as it is pointed out by Willis [16], ”The broad conclusion is that an ”effective medium” description of a composite medium provides a reasonable approximation for its response, so long as the predicted ”effective wavelength” is larger than two periods of microstructure – say at least 2.5”. This is confirmed recently on the example of Mindlin's microelasticity theory [17]. It is worth therefore to build tools for the analysis of the interaction between layers and waves with the shorter wavelength. The natural choice for such tools is provided by numerical methods due to their flexibility and universality. However, we need to be insured in the accuracy and stability of them. It is well known that numerical simulation of wave propagation even in a homogeneous solid bar under shock loading is under discussion so far both in the context of finite volume [18,19] and finite element methods [20–23]. This is why two different numerical methods – finite element method and finite volume method – are applied in the paper for the simulation of a pulse propagation in a slender heterogeneous solid bar. The pulse propagation is preferable from the practical point of view [24], while theoretically only the behavior of dispersion curves is of interest [25–27] e.g.

It should be repeated, following Zohdi [28] that ”solutions to partial differential equations, of even linear material models, at infinitesimal strains, describing the response of small bodies containing a few heterogeneities are still open problems”. Fortunately, the analytical solution is constructed specifically for the considered test problem by means of the Laplace transform technique [29].

The objective of the paper is to demonstrate the influence of the composition of alternating layers on the dispersion of a short pulse and to compare results of simulation obtained by two different numerical methods with the analytical solution of the problem.

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In this paper, we consider the propagation of a finite pulse, the length of which is comparable with the size of heterogeneities. The dispersion of the pulse is provided by the wave reflection and transmission in periodic layered structure where each layer is dispersionless. It is clearly demonstrated that strong dispersion effects depend not only on the size of heterogeneities but also on their mutual position.

2. Formulation of the problem

We consider wave propagation in a bar of a constant cross section. The motion is assumed being one-dimensional and considered within the linear theory of elastodynamics [30, e.g.] It is governed by the balance of linear momentum, which in the absence of body forces has the form

$$\rho\frac{\partial v}{\partial t}-\frac{\partial \sigma}{\partial x}=0, \tag{1}$$

where ρ is the matter density, v is the particle velocity, σ is the one-dimensional Cauchy stress. In the linear elasticity the Cauchy stress obeys the Hooke law $\sigma = E\,\varepsilon$,where E is the Young modulus and ε is the one-dimensional strain.. The wave speed in a bar is given for one-dimensional case by $c=\sqrt{E/\rho}$, therefore, the Hooke law has the following form

$$\sigma=\rho c^2\varepsilon. \tag{2}$$

The strain and velocity are related by the compatibility condition

$$\frac{\partial \varepsilon}{\partial t}=\frac{\partial v}{\partial x}. \tag{3}$$

In terms of the displacement, the balance of linear momentum is represented in the form of the wave equation

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}, \tag{4}$$

since the displacement u is connected to the strain and particle velocity by

$$v=\frac{\partial u}{\partial t},\quad \varepsilon=\frac{\partial u}{\partial x}. \tag{5}$$

It should be noted that the material parameters ρ and c are distinct in different parts of the bar. However, they keep constant values for each computational cell in numerical methods for the bar with a piecewise constant distribution of material parameters such as Young modulus and matter density.

It is assumed that the bar is occupied the interval $0\leqslant x\leqslant L$. Initially, the bar is at rest. The left end of the bar is loaded by the pulse, the shape of which is formed by an excitation of the stress for a limited time period (Fig. 1). Then the stress at the left end is zero. The right end of the bar is fixed.

For convenience, the bar is divided into three parts. The left and the right parts of the bar are supposed to be homogeneous and made from the same stiff material. The central part of the bar contains inhomogeneity provided by inclusions of a more soft materials (see Fig. 2). The solution of system of Eqs. (1)–(3) or Eq. (4) satisfying formulated initial and boundary conditions is obtained by means of analytical and numerical methods in the following sections.

3. Analytical solution

To verify the correctness and the accuracy of numerical results presented in Section 4, the analytical solution of the above described problem was derived. The main idea of the analytical procedure is based on the fact that the final solution for a bar with a piecewise constant distribution of material properties can be constructed from the solutions derived for each of homogeneous parts of the bar combined through the boundary conditions formulated at their interfaces.

It is clear that the propagation of longitudinal waves in arbitrary ith homogeneous part is formally described by the same equation as (4). The solution for such particular problem with general boundary conditions can be simply derived based on the solution presented in [30]. Applying the Laplace transform in time [29] to (4) with zero initial conditions one obtains a simple ODE the solution of which can be written as

$$\bar{u}_i(x_i,p)=C_{1,i}(p)\sinh\left(\frac{px_i}{c_i}\right)+C_{2,i}(p)\cosh\left(\frac{px_i}{c_i}\right), \tag{6}$$

where p is a complex number. The variable x_i in (6) represents a local coordinate defined for the ith part of the bar, the constant c_i is the wave speed in this part and the function $\bar{u}_i(x_i,p)$ denotes the Laplace transform of the corresponding displacement $u_i(x_i,t)$. The unknown complex functions $C_{1,i}$ and $C_{2,i}$ can then be determined through the boundary conditions of the problem and through the conditions of displacement and stress continuity formulated for each interface between two parts with different material properties. It leads to a system of algebraic equations in complex domain. Substituting its solution into (6) one obtains the final solution of the problem in Laplace domain.

The last step of the analytical procedure consists in the inversion of previously mentioned formulas back to time domain. It can be done analytically by means of the residue theorem in this case or numerically by using a suitable algorithm. Given the low computational demands and the versatility, the latter approach was used in this work. In particular, an algorithm based on FFT and Wynn's epsilon accelerator was applied to manage the inverse Laplace transform problem. As proved in [29], this algorithm is effective and robust and it gives very precise results for various problems of elastodynamics. The analytical results for specific study cases are presented together with the numerical solutions in Section 5.

4. Numerical procedures

Application of numerical methods suggests a discretization in space and time. For this purpose, the interval $0\leqslant x\leqslant L$ is divided into N elements of the same size. The state of each element is described differently in distinct methods. In this paper, we compare results obtained by the finite element method (FEM) and the finite volume method (FVM) in case of explicit approaches of these methods.

4.1. Finite element method and explicit time integration

In this section, we shortly remind the basic of the finite element method in the one-dimensional case for linear elastodynamics. Spatial discretization of elastodynamics problems by the finite element method leads to the matrix form [31]

$$\mathbf{M}\ddot{\mathbf{d}}(t)+\mathbf{K}\mathbf{d}(t)=\mathbf{F}(t),\quad t\in[0,T], \tag{7}$$

$$\mathbf{d}(0)=\mathbf{d}_0, \tag{8}$$

$$\dot{\mathbf{d}}(0)=\dot{\mathbf{d}}_0. \tag{9}$$

Here $\mathbf{M}$ denotes the mass matrix, $\mathbf{K}$ marks the stiffness matrix, $\mathbf{F}$ is the time-dependent load vector, $\mathbf{d}$, $\dot{\mathbf{d}}$ and $\ddot{\mathbf{d}}$ contain nodal variables, namely, displacements, velocities, and accelerations, respectively, t is the time and dots denote time derivatives. Initial values for displacements and velocities are denoted by $\mathbf{d}_0$ and $\dot{\mathbf{d}}_0$. The initial acceleration vector should satisfy the equation of motion at the time t_0: $\mathbf{M}\ddot{\mathbf{d}}_0+\mathbf{K}\mathbf{d}_0=\mathbf{F}_0$.

Practically, the stiffness matrix $\mathbf{K}$ and the mass matrix $\mathbf{M}$ are

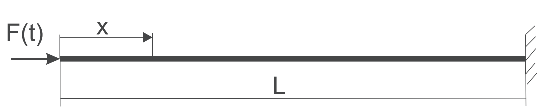

Fig. 1. A scheme of the test problem - a pulse loaded free-fixed bar.