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Analysis of effective elastic properties for shell with complex geometrical shapes

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ABSTRACT

The manuscript offers a methodology to solve the local problem derived from the homogenization technique, considering composite materials with generalized periodicity and imperfect spring contact at the interface. The general expressions of the local problem for an anisotropic composite with perfect and imperfect contact at the interface are derived. The analytical solutions of the local problems are obtained by solving a system of partial differential equations. In order to validate the model, the effective properties of the structure presented in the literature are obtained as particular cases. The solution of the local problem is used to extend the study to more complex structures, such as, wavy laminates shell composites with imperfect spring type contact at the interface. Also, the results are compared with the results for perfect and imperfect contact models available in the literature.

1. Introduction

Smarts materials composites present great potential for applications in aerospace, textile and bioengineering industries [1,2]. The development of new technologies in these areas has brought an increase in the use of composite materials and this in turn has brought the expansion and improvement of mathematical and computational methods. One of the main objective of the mathematical and computational methods is the calculation of the effective properties (elasticity, conductivity, etc.) [3–6]. The most common mathematical methods used to compute the effective properties include finite elements method (FEM) [7], Fourier series [8] and multi-scale asymptotic homogenization methods [9–11]. Some authors have used discrete singular convolution method (DSC) for the free vibration analysis of rotating conical shells [12].

Multilayered shells are the most popular composite structures due to their good mechanical properties [13]. Many authors have focused their work on the influence of the geometrical structure of the multilayered composite [14–16]. Also, it have been considered different specific structures, as cylindrical [17,18], spherical [19,20] or truncated conical shell [21]. On the other hand, important studies have been developed in order to see the influence of the contact behavior in the interface of the components on the global properties of the composite [22–24]. The imperfect spring type contact is one of the most widely studied problems. Many authors have been modeling the imperfect contact on fibrous composites with specific geometrical characteristics [25–27].

Many studies have focused their investigation to particular cases of the properties of the composite elements. The most common components are considered isotropic due to its wide appearance in problems of physics and the mechanics of solids [28]. On the other hand, some authors have extended the study of the composite structures to other types of materials (orthotropic, monoclinic, etc). In [10] the asymptotic homogenization method was used to find the effective elastic properties of composite with monoclinic components. According to [29], the DSC reports accurate results for the solution of problems considering orthotropic laminated canonical and cylindrical shells. In [30,31], the authors study the stability of a cylindrical shell composite with components of ceramic, functionally graded materials (FGM) and metal layers. these studies consider the thickness variation for the FGM layer. Some models have presented the thickness variation of the layers as a parametric function of the coordinates [32].

In this contribution, the material coefficients of an elastic

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composites are assumed to be rapidly oscillating and periodic functions of a curvilinear coordinates system. The two scales asymptotic homogenization method is used to find the homogeneous problem associated to the equilibrium problem of the system [20,32]. This work gives an approach to analyze the heterogeneous elastic problem in curvilinear structures with general anisotropy, and perfect/imperfect contact at the interface. During the homogenization process, the general expression of the local problems is obtained, considering an generalized periodic anisotropic structure. In previous works, the methods used to solve the local problems were restricted to structures with generalized periodicity but considering perfect contact at the interface [33] or to rectangular laminated composites and isotropic components [11]. As an extension of these contributions, a methodology to solve the local problem for a composite with generalized periodicity, imperfect spring type contact at the interface and anisotropic components is presented. The analytical expression of the local functions are given as a solution of linear equations. In order to validate the present approach, the effective coefficients reported in [10] for a "Chevron" structure with perfect contact at the interface are obtained as special case.The effective coefficients reported in [33] are compared with the results obtained for the imperfect contact case (spring type). As an extension of [34], the effective coefficients of three dimensional wavy laminate composite with imperfect contact at the interface are derived.

The paper is organized as follows. In Section 2, the asymptotic homogenization method is used to derive the general expression of the local problem and the interface conditions. The effective coefficient of a laminate shell composite is introduced in Section 3, where the geometry of the structure is described by a function $\varrho: \mathbb{R}^3 \to \mathbb{R}$, [33]. Also, the local problem for anisotropic components of the composite with perfect contact at the interface is obtained as a system of linear equations. In Section 4, the local problem is extended to the case of imperfect contact at the interface (spring type) and the system of partial differential equations associated to the local problem is solved. Finally, the Sections 5, 6 illustrate some examples and applications of the described methodology.

2. Asymptotic homogenization method for linear curvilinear elastic problem

In [32,34], the equilibrium elastic problem for a curvilinear composite structure $\Omega = \Omega_1 \cup \Omega_2$, bounded by the surfaces S_1, S_2, is studied. The general expression for the imperfect contact case is given by

$$\sigma^{ij}|_j + f^i = 0, \quad \text{in} \quad \Omega, \tag{1}$$

with boundary conditions

$$u_i = u_i^0 \quad \text{on} \quad S_1, \quad \sigma^{ij}n_j = S_0^i \quad \text{on} \quad S_2, \tag{2}$$

and interface conditions

$$\sigma^{ij}n_j = K^{ij}[[u_j]], \quad \text{on} \quad \Gamma, \tag{3}$$

$$[[\sigma^{ij}n_j]] = 0, \quad \text{on} \quad \Gamma. \tag{4}$$

Here $(\cdot)|_j$ denotes the contravariant derivative, f^i is the vector of the body forces, u_i is the displacement vector, n_j is outward unit normal vector of the surface S_2 or Γ and u_i^0 and S_i^0 are the prescribed values of the displacement and the stress in S_1 and S_2, respectively. The surface Γ is the interface between the two components of the composite. The matrix $\mathbf{K} = [K^{ij}]$ characterizes the imperfect contact in Γ and the order of $\mathbf{K}$ is $O(\varepsilon^{-1})$ and $[\![\cdot]\!] = (\cdot)^{(2)}$- $(\cdot)^{(1)}$ denotes the jump at the interface Γ. In particular case when the components of $\mathbf{K}$, $K_{ij} \to \infty$, the problem (1)–(4) reduces to the perfect contact case at the interface.

In order to derive the expression of a homogenized problem associated to (1)–(4), the two-scales asymptotic homogenization method (AHM) is used. In [32,33], a methodology to derive the expression of the following local problems is shown,

$$(\varrho_{q,j}C^{ijlk} + \varrho_{p,n}C^{ijmn}N^{lk}_{m|p}\varrho_{q,j})_{|q} = 0, \quad \text{on} \quad \mathbf{Y} = \mathbf{Y}_1 \cup \mathbf{Y}_2, \tag{5}$$

where $\mathbf{Y}$ is the unit cell, $\mathbf{Y}_1$, $\mathbf{Y}_2$ are the components of the unit cell and $\boldsymbol{\varrho} = (\varrho_1, \varrho_2, \varrho_3)$ is the function that described the geometry of the composite.

In [34], the two-scales asymptotic homogenization method is extended to the imperfect contact case and the following general expression of the imperfect spring type interface condition for N_m^{lk} was introduced

$$[\varrho_{q,j}C^{ijlk} + \varrho_{p,n}C^{ijmn}N^{lk}_{m|p}\varrho_{q,j}](-1)^{\alpha+1}n_j = (-1)^{\alpha}K_{ij}[[N_j^{lk}]], \text{ on } \Gamma = \overline{\mathbf{Y}}_1 \cap \overline{\mathbf{Y}}_2, \tag{6}$$

where $K_{ij} \equiv K_{ij}(\varepsilon)$.

Finally, solving the local problem (5)–(6), the general expression of the homogenized problem is

$$(C_e^{ijkl}v_{k,l})|_j + f^i = 0, \tag{7}$$

$$v_i = u_i^0 \quad \text{on} \quad S_1, \quad (C_e^{ijkl}v_k|_l)n_j = S_i^0 \quad \text{on} \quad S_2, \tag{8}$$

where the effective coefficient $\boldsymbol{C} = [C_e^{ijkl}]$ has the following expression by components [32]

$$C_e^{ijkl}(x) = \left\langle C^{ijkl} + C^{ijmn}\varrho_{p,n}\frac{\partial N_m^{kl}}{\partial y_p}\right\rangle. \tag{9}$$

In the following sections, different techniques are presented in order to solve the local problems (5) and (6) for perfect and imperfect spring contact type case at the interface.

3. Effective coefficient of a generalized stratified periodic composite with perfect contact condition

Consider a stratified laminated shell composites, where the periodicity (stratified) function $\boldsymbol{\varrho}$ has the property: $\varrho: \mathbb{R}^m \to \mathbb{R}^1$ with $m = 2, 3$ [33].

Now we consider the case when the elastic tensor $\boldsymbol{C} \equiv \boldsymbol{C}\left(\frac{\varrho(x)}{\varepsilon}\right)$, and the stratified function $\varrho: \mathbb{R}^3 \to \mathbb{R}$, i.e. $\varrho \equiv \varrho(x_1, x_2, x_3)$. Substituting this expression of ϱ into (9) and using the Voigt notation, the following equation can be obtained

$$\begin{aligned} C_e^{ab} = \Bigg\langle C^{ab} &+ \left(C^{a1}\frac{\partial\varrho}{\partial x_1} + C^{a6}\frac{\partial\varrho}{\partial x_2} + C^{a5}\frac{\partial\varrho}{\partial x_3}\right)\frac{\partial N_1^b}{\partial y} \\ &+ \left(C^{a6}\frac{\partial\varrho}{\partial x_1} + C^{a2}\frac{\partial\varrho}{\partial x_2} + C^{a4}\frac{\partial\varrho}{\partial x_3}\right)\frac{\partial N_2^b}{\partial y} \\ &+ \left(C^{a5}\frac{\partial\varrho}{\partial x_1} + C^{a4}\frac{\partial\varrho}{\partial x_2} + C^{a3}\frac{\partial\varrho}{\partial x_3}\right)\frac{\partial N_3^b}{\partial y}\Bigg\rangle. \end{aligned} \tag{10}$$

The Eq. (10) is a generalization of the results presented in [33,11] (for instance, see formula (3.35) in [33]).

3.1. Local problems

In this section, the local problem for a perfect contact case is solved, i.e. $K^{ij} \to \infty$ in (6). From (5), the following problems for the local functions $\partial N_j^a/\partial y$, where $a = 1, 2, 3, 4, 5, 6$ and $j = 1, 2, 3$ are derived in the Voigt's notation [10],

$$\frac{\partial}{\partial y}\left[b_1^a + D_{11}\frac{\partial N_1^a}{\partial y} + D_{12}\frac{\partial N_2^a}{\partial y} + D_{13}\frac{\partial N_3^a}{\partial y}\right] = 0, \tag{11}$$

$$\frac{\partial}{\partial y}\left[b_2^a + D_{21}\frac{\partial N_1^a}{\partial y} + D_{22}\frac{\partial N_2^a}{\partial y} + D_{23}\frac{\partial N_3^a}{\partial y}\right] = 0, \tag{12}$$

$$\frac{\partial}{\partial y}\left[b_3^a + D_{31}\frac{\partial N_1^a}{\partial y} + D_{32}\frac{\partial N_2^a}{\partial y} + D_{33}\frac{\partial N_3^a}{\partial y}\right] = 0, \tag{13}$$

where