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# Fracture plane based failure criteria for fibre-reinforced composites under three-dimensional stress state

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ABSTRACT

Fracture plane based failure criteria for fibre-reinforced composite materials under three-dimensional stress state are presented. The failure function is taken as a polynomial expansion in terms of the stress components on fracture plane, which provides a general mathematical technique for constructing Mohr's fracture hypothesis-based criteria. The polynomial expansion is then truncated at the quadratic terms to approximately describe the failure function. Besides the basic strengths of UD laminates, the fracture plane angles under transverse tension/compression and pure shear are introduced to calibrate the failure criteria, since Mohr's concept can be described completely and exactly only by both the basic strengths and the fracture plane angles. According to experimental evidences, the interaction between matrix-dominated and fibre-dominated failure modes is also considered in the present study. No empirical or artificially defined parameters are included in the criteria. Experimental verification for different kinds of unidirectional composites under various stress states demonstrates that the proposed failure criteria have a good predictive ability.

## 1. Introduction

The development of physically based failure criteria for reliable predictions of ply damage mechanisms in fibre-reinforced polymer (FRP) composites has been the focus of investigations for many years. The early promising phenomenological failure criteria that were able to distinguish fibre and matrix failure modes were proposed by Hashin in 1980 [1]. Even though many failure criteria [2,3] followed the yield criteria of von Mises or Hill which actually were developed for ductile materials, Hashin's criteria, based on Mohr's failure theory [4], stated that failure would be exclusively caused by the stresses acting on the material fracture plane. However, some simplifications and assumptions were made to avoid adding the term of the fracture plane angle in Hashin's quadratic approximation, due to the large computational effort and complicated computational process required in the calculation of the angle of an inclined fracture plane [1]. Nowadays Hashin's criteria are well recognized, but the stress interactions predicted do not always fit well with the experimental results, especially in the case of matrix or fibre compression [5].

Following the Hashin's idea of introducing the concept of the fracture plane into failure criteria, the fracture plane based failure theory was further developed by Puck and co-workers [6–8], and has been successfully proven its predictive capability. In the first and second World-Wide Failure Exercises (WWFE-I and WWFE-II) [9–11], which were held to evaluate the performance of current leading failure criteria for predicting the response of FRP composites, Puck's failure theory was ranked quite highly. For instance, when dealing with in-plane failure under biaxial loads, it was found to be one of the five leading failure theories among all the nineteen participating theories in WWFE-I. Thus, Puck's theory was recommended to researchers and design communities by the organizers [12], in view of its good predictive capability and solid foundation for phenomenological understanding of failure. Afterwards, further development was undertaken by Davila et al. [5], Pinho et al. [13,14] and Camanho et al. [15] with additional consideration regarding failure behaviour in the fibre direction.

Despite its good performance, the application of Puck's failure theory is restricted by a relatively large number of model parameters. Besides fracture resistances, four inclination parameters ($p^t_{\perp\parallel}$, $p^t_{\perp\perp}$, $p^c_{\perp\parallel}$ and $p^c_{\perp\perp}$) were introduced to describe the master fracture body thoroughly [8]. Only parameters $p^t_{\perp\parallel}$ and $p^c_{\perp\parallel}$ can be experimentally determined by the fracture curve under combined transverse normal and longitudinal shear stresses. However, there would be no experimentally feasible scheme to measure the other two parameters $p^t_{\perp\perp}$ and $p^c_{\perp\perp}$. Accordingly, the recommended values of these parameters were given

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**Nomenclature**

*Abbreviations*

UD unidirectional
GFRP glass-fibre-reinforced plastic
WWFE World-Wide Failure Exercises
MF matrix fracture
MFT matrix tension fracture
MFC matrix compression fracture
FRP fibre-reinforced polymer
CFRP carbon-fibre-reinforced plastic
3D three dimensional
FF fibre fracture
FFT fibre tension fracture
FFC fibre compression fracture

*Coordinate systems*

1-2-3 coordinate system of a lamina, 1 being fibre-parallel
l-n-t coordinate system of an action plane, l being fibre-parallel

*Superscripts*

t tension
m matrix
mt matrix failure under tension
mc matrix failure under compression
c compression
f fibre
fm fibre and matrix failure modes

*Subscripts*

fr fracture
fp fracture plane

*Characters*

$\sigma_n$ normal stress on a potential fracture plane
$\tau_{nt}$, $\tau_{nl}$ transverse and longitudinal shear stresses on a potential fracture plane
$\sigma_1$, $\sigma_2$, $\sigma_3$ normal stresses related to the local lamina coordinate system
$\tau_{12}$, $\tau_{13}$, $\tau_{23}$ shear stresses related to the local lamina coordinate system
$\theta$ angle between thickness direction and a parallel-to-fibre section plane
$\theta_{fp}$ angle of the fracture plane
$\theta_{fp}^{c}$ angle of the fracture plane under pure transverse compression
$X_t$, $X_c$ tensile and compressive strength of the lamina parallel to fibre direction
$Y_t$, $Y_c$ tensile and compressive strength of lamina transverse to fibre direction
$S_{12}$, $S_{13}$ longitudinal shear strength of the lamina
$S_{12}^{f}$, $S_{13}^{f}$ shear strength for shear-driven fibre breakage on the perpendicular-to-fibre plane
$F$ failure function
$A_1^{m}$, $A_1^{mt}$, $A_1^{mc}$ coefficients corresponding to linear terms of $\sigma_n$ in MF, MFT and MFC criteria
$A_2^{m}$, $A_2^{mt}$, $A_2^{mc}$ coefficients corresponding to quadratic terms corresponding to quadratic terms of $\sigma_n$ in MF, MFT and MFC criteria
$B_1^{m}$, $B_2^{m}$ coefficients corresponding to linear and quadratic terms of $\tau_{nl}$ in MF criteria
$C_1^{m}$, $C_2^{m}$ coefficients corresponding to linear and quadratic terms of $\tau_{nt}$ in MF criteria
$C_2^{mt}$, $C_2^{mc}$ coefficients corresponding to quadratic terms of $\tau_{nt}$ in MFT and MFC criteria
$D_{12}$, $D_{13}$, $D_{23}$ coefficients corresponding to terms of $\sigma_n\tau_{nl}$, $\sigma_n\tau_{nt}$ and $\tau_{nl}\tau_{nt}$ in MF criteria
$A_1^{f}$, $A_2^{f}$ coefficients corresponding to linear and quadratic terms of $\sigma_1$ in FF criteria
$B_2^{f}$ coefficient corresponding to the quadratic term of $\tau_{12}$ in FF criteria
$C_2^{f}$ coefficient corresponding to the quadratic term of $\tau_{13}$ in FF criteria
$I^{fm}$ coefficient corresponding to the coupling term of $\sigma_1\sigma_n$ in the criteria for considering interaction between fibre and matrix failure modes
$p_{\perp\parallel}^{t}$, $p_{\perp\parallel}^{c}$ inclination parameters, employed by Puck's criteria, of ($\sigma_2$, $\tau_{12}$)-fracture curve for the ranges $\sigma_2 > 0$ and $\sigma_2 < 0$, respectively
$p_{\perp\perp}^{t}$, $p_{\perp\perp}^{c}$ inclination parameters, employed by Puck's criteria, of ($\sigma_n$, $\tau_{nt}$)-fracture curve for the ranges $\sigma_n > 0$ and $\sigma_n < 0$, respectively

through calibration with limited experimental results that are mainly from two experimental projects [16]. Even so, the applicability of such values is still required to be demonstrated by a great number of materials, loading conditions, specimen configurations, etc. For example, as mentioned in [17], the value of $p_{\perp\perp}^{c}$ ($\approx$0.38), corresponding to a known fracture angle ($\approx$53°) for composites subjected to transverse compression, is not included in the recommended interval (i.e. {0.20 ~ 0.30}) of Puck's criteria. Additionally, there is no way to measure the transverse shear fracture resistance for brittle fibre reinforced composites directly and can only be evaluated by Puck's fracture condition.

It should be mentioned that stresses in fibre direction actually have influence on the matrix fracture, even though they do not act on the potential fracture plane [16–18]. This may be attributed to localized micro-damages of composite materials (single fibre fracture, micro-cracks in the matrix and debonding of the fibre matrix interfaces). In order to characterize the interactions of micro-damages, a degradation factor is employed by Puck to consider a reduction of the fracture resistance on the macro level. The major drawback, however, is the lack of reliable experimental or mathematical methods to obtain the specific value of that factor.

The present work tries to propose interactive stress-based failure criteria for FRP composites under 3D stress state, in order to avoid the use of the parameters mentioned above. Motivated by Mohr's fracture hypothesis, the failure functions, which are expressed in terms of stresses acting on the fracture plane, are derived upon the coordinate system of an action plane. Also, the interaction of different failure modes is described. All the unknown coefficients in the failure functions can be determined by the conventional UD-lamina level strength properties and the critical fracture angle for characterizing the orientation of a fracture plane caused by transverse compression. It is worth noting that if Mohr's fracture hypothesis is obeyed, the fracture angle may be employed as a known parameter to characterize the uniaxial failure of a UD laminate together with basic strengths, since only by both the fracture angle and the failure stresses, can Mohr's fracture hypothesis be described faithfully and completely. The predicted failure envelopes are compared with the measurements in biaxial, off-axis tension and tri-axial experiments to validate the present