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Inverse estimation for unknown fouling-layer profiles with arbitrary geometries on the inner wall of a forced-convection duct

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ABSTRACT

In this study, a conjugate gradient method based inverse algorithm is applied to estimate the unknown fouling-layer profile on the inner wall of a duct system using simulated temperature measurements taken on the duct wall. The temperature data obtained from the direct problem are used to simulate the exact temperature measurements. Results show that an excellent estimation on the fouling-layer profile can be obtained for the case without separation bubble. The predictive accuracy, however, slightly deteriorates when there is a separation bubble in the duct flow. The technique presented in this study can be used in a warning system to call for maintenance when the thickness of fouling exceeds a pre-defined criterion.

1. Introduction

Inverse methods have recently been applied to various engineering problems. A great number of applications of inverse methods are continuously being proposed for different technical fields [1–3]. Despite the ill-posed nature of inverse problems, the solutions of these problems are important for theoretical studies and measurement techniques, especially in cases where measurement is difficult, instruments for measurement are expensive, or the measurement process to directly measure certain physical quantities is complicated. Under these circumstances, a satisfactory estimated result can be easily obtained by using a numerical method and some simple instruments, for examples, using thermocouples to measure the inner wall temperatures of burners, the temperatures of cutting tool tips, and so on. Among those different engineering applications, one of the most important applications is on a heat exchanger, a crucial element found in many engineering devices. A fundamental characteristic of a heat exchanger is the phenomenon of conjugate heat transfer which involves an interaction between the conduction of the solid wall material and the convection of the fluid flowing over that wall. The problems of conjugate heat transfer are very important and have already been examined by a number of researchers [4–6]. Many other important engineering devices also involve conjugate heat transfer problems such as flows over fins. In this case, valuable design information can be obtained by simultaneously analyzing the conduction in the fin and the convection in the fluid. For conjugate heat transfer in thick-walled pipes or ducts, the boundary conditions imposed at the external surface are different from those which exist at the internal surface. In this situation, the thermal boundary conditions existing at the internal surface are not known a priori, and hence, the energy equations must be solved under the conditions of temperature and heat flux continuity.

One of the very interesting topics of inverse problems, attracting a lot of attention in recent years, is the technique of inverse geometry problem (or shape identification problem). The applications of shape identification problem have been widely used in various industrial fields, for examples, the prediction of frost thickness in refrigeration systems, the prediction of the geometry of blast furnace inner wall, the prediction of crevice and pitting in furnace wall, and the optimization of geometry [7]. In the past, there have been many researchers devoted to the study of inverse geometry problems using a variety of numerical methods. Huang and Chen [8] developed a modified model to estimate the outer boundary configurations of a multiple region domain without confining the search directions. Park and Shin applied the coordinate transformation technique with the adjoint variable method to a shape identification problem in determining unknown boundary configuration for heat conduction systems [9] and natural convection systems [10]. Divo et al. [11] used the genetic algorithm and a singular superposition technique to detect the unknown sphere cavity in a 3D inverse geometry problem. Kwag et al. [12]

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Nomenclature

$H1$	the height of the inner wall in the duct (m)
H_2	the height of the outer wall in the duct (m)
h	convection heat transfer coefficient (W m^{-2} K^{-1})
J	functional
J'	gradient of functional
k	thermal conductivity (W m^{-1} K^{-1})
L	length of the duct (m)
M	total number of measuring positions
p	direction of descent
T	temperature (K)
T_{in}	inlet temperature (K)
T_∞	ambient temperature (K)
u	fluid velocity in the x-direction (m s^{-1})
v	fluid velocity in the y-direction (m s^{-1})
x	spatial coordinate (m)
Y	measurement temperature (K)
y	spatial coordinate (m)
Δ	small variation quality
α	thermal diffusivity (m^2 s^{-1})
β	step size
γ	conjugate coefficient
η	very small value
λ	variable used in adjoint problem
ν	fluid kinematic viscosity $\nu = \mu/\rho$ (m^2 s^{-1})

Superscripts

K	iterative number

Subscripts

$s1$	for fouling layer
$s2$	for duct wall material
f	fluid
s	solid

followed a new algorithm to estimate the phase front motion of ice in a thermal storage system. Recently, Su and Chen [13] utilized the reversed matrix method with both the linear least-squares error method and the concept of virtual area for a shape identification problem to identify the geometry of inner wall in a furnace. Among those studies, it can be noted that there have been only very few on shape identification problems involving conjugate heat transfer which is commonly encountered in heat exchanger problems.

The performance of a heat exchanger usually deteriorates with time as a result of the accumulation of deposits on heat transfer surfaces. The layer of deposit (fouling) represents additional thermal resistance to heat transfer and causes the heat transfer rate of the heat exchanger to drop. In addition, the fouling could narrow the flow channel and result in an increase in pumping power, which in turn consumes more energy. Since fouling is often formed on the inner wall of a heat exchanger duct, it is difficult to obtain the exact configuration of the fouling layer, especially if the duct is very long. The objective of the present inverse geometry problem is to estimate the unknown irregular fouling profile on the inner wall of a duct system, which involves conjugate heat transfer, based on the simulated temperature measurements taken within the duct wall. This technique can be used in a warning system to call for maintenance when the thickness of fouling exceeds a pre-defined criterion. In the analysis, we assume the variation in the slopes of the fouling-layer profile is large, thus the duct flow will never be fully developed, and there is a possibility of forming some localized separation bubbles. Therefore, the flow field cannot be specified but needs to be solved by the Navier-Stokes equations, which have to be incorporated into the inverse procedure. In this, the Navier-Stokes equations are coupled with the inverse algorithm through the perturbation of fouling-layer profile. That is, as the fouling-layer profile is changed after an inverse iteration, a new flow field needs to be solved by the Navier-Stokes equations because the boundary of the flow domain also changed. Then the updated flow field affects the temperature distributions both in solid and fluid materials via the mechanism of conjugate heat transfer, hence altering the course of the inverse algorithm and, in turn, resulting in yet another new fouling-layer profile. The iteration cycle then goes on and on until a convergence criterion is met. Theoretically, the current inverse method is able to cope with arbitrary fouling profiles. Here, we employ the conjugate gradient method (CGM) [14–16] and the discrepancy principle [17] to the inverse geometry problem to determine the fouling-layer configuration in the duct system. The conjugate gradient method with an adjoint equation, also called Alifanov's iterative regularization method, belongs to a class of iterative regularization techniques, which mean the regularization procedure is performed during the iterative processes, thus the determination of optimal regularization conditions is not needed. On the other hand, the discrepancy principle is used to terminate the iteration process in the conjugate gradient method.

2. Analysis

2.1. Direct problem

To illustrate the methodology of developing expressions for use in determining the unknown irregular fouling profile $f(x)$, on the inner wall of a duct flow, the following steady-state heat transfer problem is considered. Fig. 1 shows a schematic representation of the considered duct flow. A duct is assumed symmetrical to the centerline. The length of the duct is L, with half heights of the inner and outer walls H_1 and H_2, respectively. The fluid temperature at the inlet is $T_{in}(y)$. After a period of operation, a layer of fouling is assumed built up on the duct's inner wall and the profile $f(x)$ of the fouling layer is assumed unknown. Then, the mathematical formulation of this steady-state forced convection heat transfer problem, covering the fouling layer, solid duct, and fluid domains, respectively, can be expressed as [18]:

Navier-Stokes equations and boundary conditions:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1a}$$

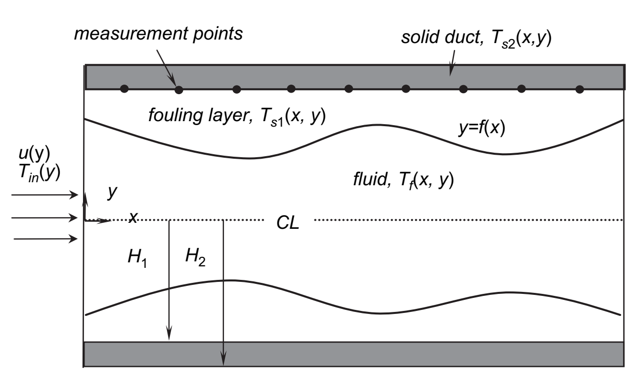

Fig. 1. Schematic of the configuration of the duct system.