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Quantifying the orthotropic damage tensor for composites undergoing damage-induced anisotropy using ultrasonic investigations

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ABSTRACT

We quantify the evolution of the general fourth order damage tensor for initially orthotropic composites undergoing damage-induced anisotropy using ultrasonic investigations. Two of the most challenging problems which arise in continuum damage mechanics are firstly the selection of variables to describe the internal damage and secondly the difficulty in modelling materials with significant initial anisotropy such as composites. This research helps advance models of anisotropic damage to overcome both these challenges. We demonstrate how to identify the directionality and magnitude of the introduced damage using experimental ultrasonic measurements of damaged elastic moduli for initially anisotropic materials. This analysis provides a robust way to validate and advance models based on continuum damage mechanics and develop phenomenological models of anisotropic damage evolution for initially orthotropic materials.

1. Introduction

Modelling and analysis of fracture propagation and progressive damage evolution are integral for damage-tolerant design in manufacturing composites as well as structural, geotechnical, mechanical, and civil engineering. Anisotropy is an important factor in producing composites with optimum utilization of the inherent strengths of the constituent materials. Manufacturing materials with optimum strength properties is important in reducing safety margins and cutting costs. With the rapidly growing advancement in material design, the assumption of scalar models of damage may not suffice and this research aims to help develop more accurate models of anisotropic damage for initially anisotropic materials such as composites or sedimentary rocks such as shale or coal.

Two of the main difficulties in considering anisotropic fourth order tensorial damage models are firstly the ability to identify the directionality of the introduced damage and secondly to perform measurements which test the predictions of the proposed models. Very few damage models have been proposed for initially anisotropic materials [1] and the correct modelling of the interaction of initial anisotropy and damage-induced anisotropy remains a much debated issue [2]. We address these challenges and quantify the relationship between a general fourth order damage tensor representing the internal damage variables using continuum damage mechanics to macroscopic, observable and empirically measured damaged elastic moduli using ultrasonic measurements. Our ultimate aim is to build mesoscale phenomenological models of damage evolution using measurements of damaged stiffness moduli from well-constrained loading experiments where the macro and mesoscale damage response should be the same or very similar. This analysis will help validate and advance anisotropic continuum damage models by providing a comparison between the predicted model values and the experimentally measured stiffness reductions using the derived relationship between these quantities detailed in this paper. We apply our analysis to experiments of stress-induced anisotropy of initially orthotropic composites undergoing stress-induced anisotropy to become orthotropic.

Ultrasonic methods have been very popular in nondestructive testing and characterization of materials and monitoring progressive damage. Nondestructive testing using ultrasonic investigations is a relatively mature field for composite materials [3–6]. Ultrasonic techniques provide fast and non-destructive methods for reliable measurement of elastic properties and their change with damage [4]. We show how ultrasonic measurements of seismic wave velocities can be used to determine the evolution of the fourth order anisotropic damage tensor characterizing the internal damage using continuum damage mechanics.

Ultrasonic techniques have been employed by several researchers (see for example [5,6,3]) to identify purely phenomenological models of anisotropic damage for composite materials. In Sections 3 and 4 we extend these models to relate the phenomenological models of experimentally measured stiffness reduction during loading given by ultrasound measurements to anisotropic fourth order tensorial damage

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models given by continuum damage mechanics. This analysis could help validate and advance anisotropic continuum damage models by providing a way to test model predictions by comparing the predicted internal damage values with the empirical damage variables given by the damaged elastic moduli measurements.

We plot the tensorial damage variables for an initially orthotropic composite undergoing damage to remain orthotropic in Section 5 using the quantitative relationship between the macroscopic, empirically observed damaged elastic moduli and the internal damage variables we derived in Section 4. In Section 5 we model the damaged-induced anisotropy of two initially orthotropic ceramic matrix composites undergoing uniaxial extension along the x_3 direction for the experimental results of Baste and Aristiégui [7] and Audoin and Baste [5]. In their experiments the orthotropic material axes are coaxial with the loading direction x_3.

In this paper we only consider damage-induced anisotropy from loading which is coaxial with the initial material axes. We consider a change in the magnitude of the initial anisotropy but not in the material axes' directions in this paper. Future work will extend this analysis to include off-axis loading where the material axes and the principal stress directions do not align, as in some of the other experimental set-ups of Baste and Aristiégui [7].

2. Modelling damage-induced anisotropy for initially orthotropic materials

Many different models of damage have been proposed using continuum damage mechanics since its inception. Many different mathematical representations for the internal damage variable have been proposed from scalar [8–10] etc; to second order tensors [11–13] etc; to fourth order tensors [14–16,13] etc and even eighth order tensors [17]. In many continuum damage models for composites several scalar damage variables are employed to represent different damage modes such as longitudinal damage (along fibres etc) or transverse damage (in matrix) (see for example: [18–21]).

Single scalar damage models are restricted to modelling isotropic damage only and have a further restriction that Poisson's ratio of the material does not change with damage. This restriction may not be physically realistic as we will show in our analysis of experimental results of anisotropic materials where Poisson's ratio changes significantly in some cases. The variation of Poisson's ratio with damage was also confirmed in our previous work [22]. Second-order tensorial anisotropic damage representation is restrictive compared to fourth-order tensorial formulation, but since its interpretation is quite simple it has been widely used for either metallic or quasi-brittle materials. In this paper we consider the most general case using the principle of strain equivalence and represent anisotropic damage using a fourth order damage tensor.

Several micromechanical approaches have been employed including effective medium theory [23–25] etc and microplane models [26–29] etc, to model the progressive degradation of anisotropic and isotropic materials. Closed form results for damage induced anisotropy in initially anisotropic materials are available for 2D problems using micromechanical approaches [24], however for 3D damage it is much more difficult. Sarout and Guéguen [23] have obtained an exact solution for a transverse isotropic rock containing cracks that run parallel to the plane of isotropy. Fabric tensors have also been related to the damage tensor in the work of Voyiadjis and Kattan [30,31]. We present an alternative phenomenological approach based on ultrasonic elastic wave velocity measurements and continuum damage mechanics. In this paper we also use ultrasonic measurements to quantify the anisotropic 3D damage in a similar approach to these micromechanical approaches. However we relate the ultrasonic measurements to the general internal fourth rank tensorial damage variables defined using continuum damage mechanics for initially orthotropic materials undergoing damage-induced anisotropy to remain orthotropic.

Instead of modelling the various damage mechanisms at the microscale level, we represent the damage indirectly by modelling the average material degradation at the mesoscale for an initially orthotropic material undergoing damage-induced anisotropy to remain orthotropic. In Sections 3 and 4 we outline the theoretical approach and relate the mesoscopic fourth rank tensorial damage variables to macroscopic ultrasonic measurements of the elastic wave velocities. In this work we focus on experiments where the macroscopic and mesoscopic damage response should be very similar before ultimate failure and rupture of the specimen. We apply our analysis to the experiments of Baste and Aristiégui [7] and Audoin and Baste [5] in Section 5. Future work will extend upon these models of orthotropic damage resulting from well-defined and constrained loading experiments which result in orthotropic damage at both the meso and macroscale level to modelling localized damage at the mesoscale level.

3. Fourth order damage tensor to characterize the internal damage

In the pioneering work of Cauvin and Testa [15] they showed using the principle of strain equivalence that in general only a fourth order tensor is needed to describe a material undergoing damage. They also showed that the actual number of independent damage parameters in such a tensor is related to the material and damage symmetry. Cauvin and Testa [15] and Jarić et al. [32] showed that the general supersymmetry requirements for the damaged elastic stiffness tensor $\widetilde{E}$ requires that $\widetilde{E}_{ijkl} = \widetilde{E}_{jikl} = \widetilde{E}_{ijlk} = \widetilde{E}_{klij}$, and this requirement places the following constraint on the damage tensor:

$$D_{ijrs}E_{rskl} - D_{klrs}E_{rsij} = 0, \tag{1}$$

$$\text{where} \quad \widetilde{E}_{ijkl} = (I_{ijrs} - D_{ijrs})E_{rskl}, \tag{2}$$

$$\text{and} \quad I_{ijrs} = \frac{1}{2}(\delta_{ir}\delta_{js} + \delta_{is}\delta_{jr}).$$

where D is the damage tensor, $\widetilde{E}$ is the damaged stiffness tensor, E is the original undamaged stiffness tensor and δ_{ij} is the Kronecker delta function. Here we note that the damage tensor is not supersymmetric like $\widetilde{E}$ but it does have the same number of independent variables as $\widetilde{E}$ [32]. Because E is supersymmetric Eq. (1) implies that D possesses minor symmetries: $D_{ijkl} = D_{jikl} = D_{ijlk}$. The above equations hold for any material symmetry of the initially undamaged and damaged material.

Here we note that we use the conventional definition of the damage tensor using continuum damage mechanics defined as: $\widetilde{E} = (I-D)$: E in Eq. (2) using the double inner product instead of a new definition of the damage tensor used by [5,6,3]. Audoin and Baste [5] originally defined the damage tensor used in these references using an additive form where: $\widetilde{E} = E - E_d$, where E_d is their damage tensor before normalization.

4. Damage induced anisotropy in initially orthotropic materials

The compliance tensor in Voigt notation for an initially orthotropic material where the model axes x, y, z are the same as the material axes is:

$$S = \begin{pmatrix} 1/E_x & -\nu_{yx}/E_y & -\nu_{zx}/E_z & 0 & 0 & 0 \\ -\nu_{yx}/E_y & 1/E_y & -\nu_{zy}/E_z & 0 & 0 & 0 \\ -\nu_{zx}/E_z & -\nu_{zy}/E_z & 1/E_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{yz} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{xz} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{xy} \end{pmatrix}$$

where E_i is the Young's modulus in direction x_i, G_{ij} is the shear modulus for coordinate planes i-j, and ν_{ij} is the Poisson's ratio in the direction of the second subscript produced by a load in the direction of the first