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Linear and nonlinear free and forced vibrations of graphene reinforced piezoelectric composite plate under external voltage excitation

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ABSTRACT

Graphene reinforcements can obviously enhance the piezoelectric properties as well as the mechanical properties of the polyvinylidene fluoride (PVDF). This paper investigates the linear and nonlinear vibration behaviors of the smart piezoelectric composite plate reinforced by uniformly and non-uniformly dispersing graphene platelets (GPLs). The effective Young's modulus is predicted by the Halpin Tsai's parallel model while the effective mass density, Possion's ratio and piezoelectric properties are calculated by the rule of the mixture. Based on the first-order shear deformation plate theory, von Karman nonlinear geometric relationship and Hamilton's principle, the governing equations of motion under different boundary conditions are derived for the smart piezoelectric composite plate. The governing equations of motion are solved to obtain the nonlinear eigenvalue equations by the differential quadrature (DQ) method. The analysis is validated by comparing with the current results of the smart piezoelectric composite plate. The effects of the GPL distribution pattern, stratification number, concentration and geometry of GPLs, plate geometry, external voltage and piezoelectric properties of GPLs as well as boundary conditions on the linear and nonlinear vibration behaviors are discussed in detail. The numerical results clearly illustrate that there exists the great potential for using GPLs in achieving smart structures with significantly improved structural stiffness.

1. Introduction

As a two-dimensional carbon and one-atom-thick layer of carbon, graphene has recently attracted considerable academic and industrial attention due to its low mass density, superior mechanical, thermal and electrical properties [1–5]. For example, the Young's modulus and ultimate strength of graphene and its derivatives can respectively reach up to 1 TPa and 130 GPa [6], and the intrinsic mobility limit is $2 \times 10^5\ \mathrm{cm^2\,V^{-1}s^{-1}}$, which exceeds the highest known materials [7]. Theories and experiments observed that the judicious incorporation of graphene or graphene platelets (GPLs) into the polymer matrix by uniform dispersion and fine bonding can significantly develop high performance composites [8–14]. Rafiee et al. [8] conducted an experimental investigation on graphene based on the epoxy nanocomposites and found that the Young's modulus of the nanocomposite increases to 131% when 0.1% weight fraction (wt%) of GPLs is added. Layek [13] reported that 124%, 97% and 121% are respectively increased in the storage modulus, stress at break and Young's modulus when 0.75% volume fraction (vol%) of graphene is distributed into the PVDF matrix. The electrical conductivity of the epoxy resin/graphite nanocomposites increases significantly by 12-order [14]. Hence, graphene-polymer nanocomposites are promising to enhance the mechanical properties for applications in aerospace, automotive and civil engineering.

With more understanding about the dynamic behaviors of GPLs reinforced composites, the GPLs reinforced structures can be better used in the engineering. Functionally graded materials (FGMs) are usually a mixture of two distinct material phases with continuously varying volume fractions of constituent materials and perform a continuous and smooth manner. Many experimental and numerical results showed that FGMs can significantly improve the resistance to the deformation and damage [15–17], and can enhance the dynamic stability of the materials and structures [18–20]. A number of important studies of the dynamic characteristics, especially the nonlinear dynamics of the FGM structures have been reported in the last two decades [21–24]. Recently, Yang and his coauthors [25] investigated the nonlinear free vibration of functionally graded polymer composite beams reinforced with graphene nanoplatelets (GPLs). They have done many valuable research works to explore the dynamic characteristics of these functionally graded structures [26–30], including bending [31–33],

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Nomenclature

a	length of the GRPC plate
b	width of the GRPC plate
h	thickness of the GRPC plate
N	total number of the GRPC plate
$\hat{\phi}$	external voltage excitation
V_{gpl}	total GPLs volume fraction in the GRPC plate
E_i	effective Young's modulus of the *ith* layer of the GRPC plate
E_M	effective Young's modulus of the PVDF matrix
E_G	effective Young's modulus of the GPL
ν_i	Poisson's ratio of the *ith* layer of the GRPC plate
ν_M	Poisson's ratio of the PVDF matrix
ν_G	Poisson's ratio of the GPL
$e_{km,i}$	Piezoelectric constant of the *ith* layer of the GRPC plate
$e_{km,M}$	piezoelectric constant of the PVDF matrix
$e_{km,G}$	Piezoelectric constant of the GPL
U	displacement component on the midplane of the plate in the X direction
V	displacement component on the midplane of the plate in the Y direction
W	Displacement component on the midplane of the plate in the Z direction
u_1	displacement field in the X direction
u_2	displacement field in the Y direction
u_3	displacement field in the Z direction
x	x-coordinate
y	y-coordinate
z	z-coordinate
t	time
Q_{ij}	plane stress-reduced stiffinesses
E_x	electric field along X direction
E_z	electric field along Z direction
ϕ	magnitude of the external electric voltage in the mid-plane of the GRPC plate
Π_s	strain energy
N_1	grid points along the ζ *axis*
C	weighting coefficients
$l_m(\zeta)$	Largrange interpolation polynomials
$\mathbf{d}$	displacement vector
ω	dimensionless natural frequency
α	times that the piezoelectric properties of the GPLs larger than these of the PVDF
a_{gpl}	length of the GPL
b_{gpl}	width of the GPL
h_{gpl}	thickness of the GPL
Δh	thickness of each individual plate
V_0	external electric voltage
V_i	GPLs volume fraction in the *ith* layer of the GRPC plate
ρ_i	mass density of the *ith* layer of the GRPC plate
ρ_M	mass density of the PVDF matrix
ρ_G	mass density of the GPL
$\boldsymbol{\varepsilon}$	strain matric of the GRPC plate
$\boldsymbol{\sigma}$	stress matrix of the GRPC plate
$\mathbf{D}$	electric displacement matrix of the GRPC plate
$\kappa_{km,i}$	dielectric constant of the *ith* layer of the GRPC plate
$\kappa_{km,M}$	dielectric constant of the *ith* layer of the PVDF matrix
$\kappa_{km,G}$	dielectric constant of the GPL
u	dimensionless form of U
v	dimensionless form of V
w	dimensionless form of W
K_s	shear correction factor
φ_x	cross rotation in the XOZ plane
φ_y	cross rotation in the YOZ plane
ζ	dimensionless form of x
ξ	dimensionless form of y
β	$\beta = \frac{\pi}{h}$
τ	dimensionless form of t
A	domain of the mid-plane of the GRPC plate
E_y	electric field along Y direction
N_x,N_y	normal resultants forces
$\bar{\phi}$	dimensionless form of ϕ
Π_k	kinetic energy
N_2	grid points along the ξ *axis*
$\mathbf{M}$	mass matrix
$\mathbf{K_L}$	linear stiffness matrix
$\mathbf{K}_{NL}$	nonlinear stiffness matrix
$\mathbf{d}^*$	vibration mode vector

buckling and postbuckling [34,35], linear and nonlinear free and forced vibrations [25,36] of GPLs reinforced composite structures through the analytical method, numerical method, finite element analysis and molecular dynamics. Their results proved that a small amount of GPL nanofillers dispersing into the polymer matrix can improve its stiffness drastically, reduce its elastic deflection and make a remarkable reinforcing effect on the buckling and postbuckling of the structures. The distribution pattern plays an essential role in the structure performances. The excellent dynamic characteristics of the piece-wise functionally graded graphene reinforced composite in the thermal environments are also reported by Shen and his group [37–41]. Kiani [42] discussed the large amplitude free vibration of the GPLs reinforced composite plate based on the isogeometric finite element.

However, there is few literature considering the vibration responses of the smart piezoelectric composite structures reinforced by graphene even the experimental studies have shown that graphene reinforcements can obviously enhance the piezoelectric, dielectric and pyroelectricity properties as well as the mechanical properties of the PVDF [13,43–45]. Abolhasani et al. [46] experimentally investigated the morphology, crystallinity, polymorphism and electrical outputs of graphene reinforced PVDF composites and demonstrated that the developed PVDF/graphene can be a potential application for the portable self-powering devices. Xu et al.'s experiments [47] observed the positive piezoconductive effect in suspended graphene layers, which highly depends on the layer number.

Herein, we investigate the linear and nonlinear vibrations of the GRPC plate in which the GPLs are uniformly and non-uniformly dispersed into the PVDF matrix. Through the first-order shear deformation plate theory and Hamilton's principle, we derive the governing equations of the motion for the GRPC plate under different boundary conditions. The governing equations of the motion are solved to obtain the linear and nonlinear eigenvalue equations by the differential quadrature (DQ) method. The effects of the GPL distribution pattern, stratification number, concentration and geometry of GPLs, plate geometry, external voltage and piezoelectric properties of GPLs as well as boundary conditions on the vibration behaviors are discussed in detail.

2. Theoretical formulation

Fig. 1 shows a multilayer GPLs reinforced piezoelectric composite plate in a Cartesian coordinate system with length a, width b and thickness h, consisting of an even number of layers N. The thickness of each individual layer is same and equals to $\Delta h = h/N$. There is an external voltage excitation $\hat{\phi}(x, y, z, t)$ loading to the GRPC plate. The piezoelectric polarized direction is along the positive z-axis. The GPL reinforcements are assumed to be uniformly disperse in each individual