International Journal of Thermal Sciences 49 (2010) 209–217

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Frequency-modulated hyperbolic heat transport and effective thermal properties in layered systems

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ARTICLE INFO

Article history:
Received 10 March 2009
Received in revised form
4 June 2009
Accepted 7 July 2009
Available online 29 July 2009

Keywords:
Hyperbolic heat conduction
Cattaneo equation
Effective thermal properties
Layered system

ABSTRACT

In this work heat transport in layered systems is analyzed using a hyperbolic heat conduction equation and considering a modulated heat source for both Dirichlet and Neumann boundary conditions. In the thermally thin case, with Dirichlet boundary condition, the well known effective thermal resistance formula is derived; while for Neumann problem only a heat capacity identity is found, due to the fact that in this case this boundary condition cannot become asymptotically steady when modulation frequency goes to zero. In contrast in the thermally thick regime, heat transport shows a strong enhancement when hyperbolic effects are considered. For this thermal regime, an analytical expression, for both Dirichlet and Neumann conditions, is obtained for the effective thermal diffusivity of the whole system in terms of the thermal properties of the individual layers. It is shown that the magnifying effects on the effective thermal diffusivity are especially remarkable when the thermalization time and the thermal relaxation time are comparable. The limits of applicability of our equation, in the thermally thick regime are shown to provide useful and simple results in the characterization of layered systems. Enhancement in thermal transport and in the effective thermal diffusivity is a direct consequence of having taken into account the fundamental role of the thermal relaxation time in addition to the thermal diffusivity and thermal effusivity of the composing layers. It is shown that our results can be reduced to the ones obtained using Fourier heat diffusion equation, when the thermal relaxation times tend to zero.

1. Introduction

Effective models have provided a useful basis for the interpretation of experimental data and understanding of heat transport in non-homogeneous systems [1]. The most of these models are based on Fourier law, which is supported by an impressive quantity of useful and successful results that show a very good agreement with experimental data for a great variety of experimental conditions [2,3]. However, it is also well known that Fourier heat diffusion law predicts an infinite velocity for heat propagation, in such a way that a temperature change in any part of the material would result in an instantaneous perturbation at each point of the sample. This inconsistency has been studied by different researchers, and a variety of models have been suggested to solve this situation. For a comprehensive account on this subject the reader is referred to the review articles of Joseph and Preziosi [4], Ozisik and Tzou [5] and the recent book by Wang et al. [6]. The origin of this fundamental problem is due to the fact that Fourier law establishes explicitly that, when a temperature gradient at time t is imposed, the heat flux starts instantaneously at the same time t. Considering that heat transport is due to microscopic motion and collisions of particles, atoms and molecules, it is straightforward to conclude that the Fourier condition on the velocity of heat transport cannot be sustained [4,7,8]. One of the simplest and accepted models [6] to solve the inconsistency of Fourier law was suggested by Cattaneo [9] and independently by Vernotte [10]. These authors incorporate the finite propagation speed of heat while retaining the basic nature of Fourier law, modifying the heat flux equation in the form:

$$\vec{J}(\vec{x}, t+\tau) = -k\nabla T(\vec{x}, t), \quad (1)$$

where $\vec{J}$ [W/m^2] is the heat flux vector, T [K] is the absolute temperature, k [W/m K] is the thermal conductivity and τ [s] is a thermal property of the medium known as the thermal relaxation time, which represents the time necessary for the initiation of the heat flux after a temperature gradient has been imposed at the boundary of the medium. Eq. (1) establishes that the heat flux does

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1290-0729/$ – see front matter
doi:10.1016/j.ijthermalsci.2009.07.005

Nomenclature

c	specific heat, J/kg K
f	frequency, Hz
F	dimensionless parameter
I	light beam intensity, W/m^2
J	heat flux, W/m^2
k	thermal conductivity, W/m K
l	thickness, m
q	complex wave number, m^{-1}
Q	positive constant, W/m^3
R	reflection coefficient
Re()	real part
S	heat source, W/m^3
t	time, *s*
T	temperature, *K*
x	spatial coordinate, m

Greek symbols

α	thermal diffusivity, m^2/s
ε	thermal effusivity, Ws$^{1/2}$/m^2 K
η	efficiency at which the absorbed light is converted into heat
θ	spatial part of the oscillatory temperature, K
Θ	positive constant, K
λ	complex parameter
μ	classical thermal diffusion length, m
ρ	density, kg/m^3
τ	thermal relaxation time, s
χ	dimensionless real parameter
ω	angular frequency, rad/s

Subscripts

ac	relative to the time-dependent temperature
amb	ambient
dc	relative to a time-independent temperature
0	relative to the semi-infinite layer
1	relative to the first finite layer
2	relative to the second finite layer

not start instantaneously, but rather grows gradually with the thermal relaxation time after the application of the temperature gradient. Conversely, τ represents the time necessary for the disappearance of the heat flux after the removal of temperature gradient [4,6].

From Eq. (1), expanding the heat flux vector in Taylor series around $\tau = 0$, and approximating at first order in τ,

$$\vec{J}(\vec{x},t) + \tau\frac{\partial \vec{J}(\vec{x},t)}{\partial t} = -k\nabla T(\vec{x},t). \tag{2}$$

The solution of this equation is given by

$$\vec{J}(\vec{x},t) = -\frac{k}{\tau}e^{-t/\tau}\int_{-\infty}^{t} e^{\xi/\tau}\nabla T(\vec{x},\xi)\mathrm{d}\xi. \tag{3}$$

This equation establishes that the heat flux vector $\vec{J}(\vec{x},t)$ at a certain time t depends on the history of the temperature gradient established in the whole time interval from $-\infty$ to t. This indicates that the heat flux has thermal memory, consequence of the finite value of the thermal relaxation time [11]. In this way, Eq. (3) predicts a dependence of the time path of the temperature gradient rather than an instantaneous response predicted by Fourier law.

Otherwise energy conservation equation is given by [2]

$$\nabla\cdot\vec{J}(\vec{x},t) + \rho c\frac{\partial T(\vec{x},t)}{\partial t} = S(\vec{x},t), \tag{4}$$

where ρ [kg/m^3] is the density, c [J/kg K] is the specific heat of the medium and the source S [W m^3] is the rate per unit volume at which the heat flux is generated. Combining Eqs. (2) and (4), the hyperbolic Cattaneo–Vernotte heat conduction equation is obtained [9,10]

$$\nabla^2 T(\vec{x},t) - \frac{1}{\alpha}\frac{\partial T(\vec{x},t)}{\partial t} - \frac{\tau}{\alpha}\frac{\partial^2 T(\vec{x},t)}{\partial t^2} = -\frac{1}{k}\left(S(\vec{x},t) + \tau\frac{\partial S(\vec{x},t)}{\partial t}\right), \tag{5}$$

where $\alpha = k/\rho c$ is the thermal diffusivity of the medium. On the left hand side of this equation, the second order time derivative term indicates that heat propagates as a wave with a characteristic speed $\sqrt{\alpha/\tau}$. Note that the first order time derivative term corresponds to a diffusive process, which is damping spatially the heat wave. Eq. (5) reduces to the parabolic heat diffusion equation (based on Fourier law) for $\tau \to 0$ or in steady-state conditions $\partial\vec{J}(\vec{x},t)/\partial t = 0$ [4].

The applicability of Cattaneo–Vernotte equation and its generalizations has been widely discussed in the literature [4,6,12–16]. It is clear that a physical system would follow the predicted hyperbolic behavior if the time scale of the heat transport phenomena analyzed is of the order of the thermal relaxation time. This quantity has been reported to be of the order of microseconds (10^{-6} s) to picoseconds (10^{-12} s) for metals, superconductors and semiconductors [7]. These small values of the thermal relaxation time indicate that its effects will not be significant if the physical time scales are of the order of microseconds or larger. In these situations Fourier approach provides adequate results. However, in modern applications such as analysis and processing of materials using ultrashort laser pulses and high speed electronic devices, the finite value of the thermal relaxation time is necessary to be considered [11–17].

One of the most interesting questions is the applicability of the hyperbolic formalism in materials with non-homogeneous inner structure, such as biological tissues and granular materials, in which several authors have claimed that they have observed hyperbolic effects with thermal relaxation times of the order of seconds [12,14]. This has generated a great controversy, because another group of authors have argued that it is enough to consider the traditional Fourier approach [18].

Recently, in the study of heat transport in nanofluids, different research groups have reported thermal conductivities much higher than the values predicted by the conventional mean field models [19,20]. These results have induced to some authors to consider that the hyperbolic equation for heat transfer could be a good option to explain the experimental data for the thermal properties of nanofluids [21]. This is due to the fact that high values of the thermal relaxation times or the presence of nanoelements could generate hyperbolic effects and consequently high thermal conductivity values for a composed system [22], because that in hyperbolic models; heat transport behaves more wave-like than in the traditional Fourier parabolic approach [4].