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Wave propagation characteristics in magneto-electro-elastic nanoshells using nonlocal strain gradient theory

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ABSTRACT

In this paper, the wave propagation in magneto-electro-elastic (MEE) nanoshells is investigated via two nonlocal strain gradient shell theories, namely, the Kirchhoff–Love shell theory and the first-order shear deformation (FSD) shell theory. By using Hamilton's principle, we derive the governing equations, which are then solved analytically to obtain the dispersion relations of MEE nanoshells. Results are presented to highlight the influences of the temperature change, external electric potential, external magnetic potential, external load, nonlocal parameter and length scale parameter on the wave propagation characteristics of MEE nanoshells. It is found that the electro-magneto-mechanical loadings can lead to the cut-off wave number at which the frequency reaches to zero.

1. Introduction

Magneto-electro-elastic (MEE) materials are known as a type of smart materials which can create magneto-electrical coupling effect when they are exposed to mechanical stresses. In converse, they can produce a strain by the application of a magneto-electrical field [1–3]. These properties make them suitable for a wide variety of applications, such as sensors, actuators, and spintronics devices, among others. Recently, MEE nanomaterials have received a great attention by the research community due to their novel mechanical, electrical, magnetic and other properties compared to their macroscopic counterparts [4–8]. The strong magnetoelectric coupling of MEE nanomaterials was observed in Fe_3O_4 nanowires [9] and multiferroic ultrathin films [10]. Narayanan et al. [11] fabricated a single nanowire multiferroic system exhibiting the room temperature magnetodielectric coupling. Tsai et al. [12] found that the increased frequency and the enhanced intensity of the tetrahedral site phonon modes were the result of the strong magnetoelastic coupling in multiferroic nanostructures.

MEE nanomaterials and their nanostructures are within the order of a nanometer. The size-dependent properties of MEE nanomaterials and their nanostructures have been observed in many experimental and atomistic simulations. Size effect on the ferroelectric phase transition in $SrBi_2Ta_2O_9$ nanoparticles was reported by Yu et al. [13] and Ke et al. [14] using thermal analysis and Raman scattering. Wang et al. [15] revealed the thickness dependent size effect on ferroelectric behavior of $BiFeO_3$ films in the polarization versus electric field hysteresis loops. Chen et al. [16] examined the size-dependent infrared phonon modes and ferroelectric phase transition in $BiFeO_3$ nanoparticles. Reddy et al. [17] studied the particle size effect in the range 10–150 nm on the magnetic properties and phase transitions in $BiFeO_3$ samples. They observed that the increase in magnetization in 12 nm particle size samples was about four times larger than that of the bulk samples. These studied indicated the importance of considering the size effect in theoretical and experimental studies of MEE nanostructures.

The theoretical studies of the size-dependent mechanical properties of nanostructures have been extensively conducted by using different higher order continuum theories. One of the most popular theories incorporating the size effect is the Eringen's nonlocal theory [18–21]. This theory can exhibit the stiffness-softening effect of the nonlocal stress field on the mechanical performances of nanostructures. It has some shortcomings due to neglecting stiffness-hardening mechanism reported in many theoretical and experimental works. The strain gradient theory [22–25] is able to predict the stiffness-hardening effect by introducing the length scale parameter. Recently, Lim et al. [26] developed the nonlocal strain gradient theory to bring both of the nonlocal parameter and length scale parameter into a single theory via the

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dynamics frame work. The nonlocal strain gradient theory can well describe the stiffening effects and the softening effects of materials at the same time.

After Lim et al.'s pioneering work, a few studies have been performed on the basis of the nonlocal strain gradient theory. Simsek [27] used the nonlocal strain gradient theory to capture the size effect on the nonlinear natural frequencies of functionally graded nanobeams. Farajpour et al. [28] proposed a new size-dependent plate model for buckling of orthotropic nanoplates. Ma et al. [29] established a nonlocal strain gradient beam model to solve the bending and buckling problems of nanobeams. Li et al. [30] utilized the nonlocal strain gradient theory to explore bending, buckling and free vibration of axially functionally graded nanobeams. Extensive studies can be further referred to the articles [31–36] for the bending, buckling, vibration and wave propagation of nanostructures. Ebrahimi et al. [37] analyzed the wave dispersion behavior of smart rotating magneto-electro-elastic nanoplates based on the nonlocal strain gradient theory. They discussed both stiffness-softening and stiffness-hardening behaviors of nanostructures. Later, they studied the wave dispersion characteristics of rotating heterogeneous MEE nanobeams [38] and rotating thermo-elastically-actuated nanobeams [39]. Aghdam et al. [40] analyzed the size-dependent buckling and postbuckling behavior of MEE composite nanoshells under the combination of the axial compressive load and electromagnetic potentials.

Based on the nonlocal strain gradient form of the Kirchhoff–Love shell theory and the first-order shear deformation (FSD) shell theory, this paper analyzes the wave propagation characteristics in MEE nanoshells subjected to thermo-electro-magneto-mechanical loadings. The governing equations are derived by using the Hamilton's principle. The dispersion relations of MEE nanoshells are obtained by solving an eigenvalue problem. Numerical results show that the nonlocal parameter, length scale parameter, temperature change, external electric potential, external magnetic potential and external load have important influence on the wave propagation characteristics of MEE nanoshells.

2. The nonlocal strain gradient theory for MEE materials

The Eringen's nonlocal theory [18–21] states that the stress field at a reference point is assumed to depend not only on the strain at the reference point but also on the strains at all other points in the whole body. The strain gradient theory [22–25] assumes that the materials must be considered as atoms with the higher-order deformation mechanism at micro/nano scale rather than just modeled them as collections of points. The nonlocal strain gradient theory takes into account both nonlocal elastic stress field and strain gradient stress field by introducing two scale parameters. For MEE nanomaterials, this theory can be written as [41]:

$$[1-(e_0a)^2\nabla^2]\sigma_{ij} = (1-l^2\nabla^2)(c_{ijkl}\varepsilon_{kl}-e_{mij}E_m-q_{nij}H_n-\beta_{ij}\Delta T), \tag{1}$$

$$[1-(e_0a)^2\nabla^2]D_i = (1-l^2\nabla^2)(e_{ikl}\varepsilon_{kl}+s_{im}E_m+d_{in}H_n+p_i\Delta T), \tag{2}$$

$$[1-(e_0a)^2\nabla^2]B_i = (1-l^2\nabla^2)(q_{ikl}\varepsilon_{kl}+d_{im}E_m+\mu_{in}H_n+\lambda_i\Delta T), \tag{3}$$

$$\sigma_{ij,j} = \rho\ddot{u}_i, \quad D_{i,i}=0, \quad B_{i,i}=0, \tag{4}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j}+u_{j,i}), E_i = -\widetilde{\Phi}_{,i}, H_i = -\widetilde{\Psi}_{,i}, \tag{5}$$

where σ_{ij}, ε_{ij}, D_i, E_i B_i, H_i and u_i are the stress, strain, electric displacement, electric field, magnetic induction, magnetic field and displacement components, respectively; $\widetilde{\Phi}$ and $\widetilde{\Psi}$ are the electric potential and magnetic potential, respectively; c_{ijkl}, e_{mij}, s_{im}, q_{ij} d_{ij}, μ_{ij}, p_i and λ_i are elastic, piezoelectric, dielectric constants, piezomagnetic, magneto-electric, magnetic, pyroelectric and pyromagnetic constants, respectively; β_{ij} and ρ are the thermal moduli and mass density, respectively; ΔT is the temperature change; e_0a and l are the nonlocal parameter and the length scale parameter describing the small-scale effect in MEE nanostructure, respectively; ∇^2 is the Laplace operator.

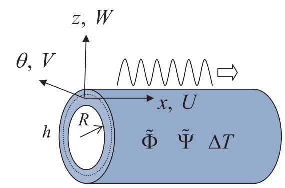

Fig. 1. Flexural wave propagation in an MEE nanoshell.

3. Wave propagation in nonlocal strain gradient MEE nanoshells

By using the nonlocal strain gradient theory for MEE materials, we will develop two nanoshell models to analyze the size-dependent wave propagation characteristics in MEE nanoshells. Fig. 1 shows an MEE cylindrical nanoshell with the radius R and thickness h subjected to an electric potential $\widetilde{\Phi}$, a magnetic potential $\widetilde{\Psi}$, an axial load P_0 and a uniform temperature change ΔT. It is assumed that (x,θ,z) is the coordinate system fixed at the midplane of the nanoshell. The MEE nanoshell is polarized along the thickness direction.

3.1. Nonlocal strain gradient Kirchhoff–Love shell model

Based on the Kirchhoff–Love shell theory, the displacements of an arbitrary point in the shell along the x-, θ- and z- axes, denoted by $u_x(x,\theta,z,t)$, $u_\theta(x,\theta,z,t)$ and $u_z(x,\theta,z,t)$, respectively, are

$$u_x(x,\theta,z,t) = U(x,\theta,t)-z\frac{\partial W(x,\theta,t)}{\partial x}, \tag{6}$$

$$u_\theta(x,\theta,z,t) = V(x,\theta,t)-z\frac{\partial W(x,\theta,t)}{\partial \theta}, \tag{7}$$

$$u_z(x,\theta,z,t) = W(x,\theta,t), \tag{8}$$

where $U(x,\theta,t)$, $V(x,\theta,t)$ and $W(x,\theta,t)$ are the displacements of a point in the midplane and t is the time.

The relations of strain and displacement can be written as

$$\varepsilon_{xx} = \frac{\partial U}{\partial x}-z\frac{\partial^2 W}{\partial x^2}, \tag{9}$$

$$\varepsilon_{\theta\theta} = \frac{1}{R}\left(\frac{\partial V}{\partial \theta}+W\right)-\frac{z}{R^2}\left(\frac{\partial^2 W}{\partial \theta^2}-\frac{\partial V}{\partial \theta}\right), \tag{10}$$

$$\gamma_{x\theta} = \frac{\partial V}{\partial x}+\frac{1}{R}\frac{\partial U}{\partial \theta}-\frac{2z}{R}\left(\frac{\partial^2 W}{\partial x\partial \theta}-\frac{\partial V}{\partial x}\right). \tag{11}$$

The distributions of electric potential $\widetilde{\Phi}(x,\theta,z,t)$ and magnetic potential $\widetilde{\Psi}(x,\theta,z,t)$ of MEE nanoshells are assumed as linear combinations of cosine and linear variations, which satisfies the Maxwell equations [42]. Then, we have

$$\widetilde{\Phi}(x,\theta,z,t) = -\cos(\beta z)\Phi(x,\theta,t)+\frac{2z\phi_0}{h}, \tag{12}$$

$$\widetilde{\Psi}(x,\theta,z,t) = -\cos(\beta z)\Psi(x,\theta,t)+\frac{2z\psi_0}{h}, \tag{13}$$

where $\beta = \pi/h$; $\Phi(x,\theta,t)$ and $\Psi(x,\theta,t)$ are the variation of the electric potential and magnetic potential at the midplane, respectively; ϕ_0 and ψ_0 are the applied external electric potential and magnetic potential, respectively.

With the aid of Eqs. (12) and (13), we can yield the electric field (E_x,E_θ,E_z)

$$E_x = -\frac{\partial \widetilde{\Phi}}{\partial x} = \cos(\beta z)\frac{\partial \Phi}{\partial x}, \tag{14a}$$