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Bloch wave filtering in tetrachiral materials via mechanical tuning

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ABSTRACT

The periodic cellular topology characterizing the microscale structure of a heterogeneous material may allow the finest functional customization of its acoustic dispersion properties. The paper addresses the free propagation of elastic waves in micro-structured cellular materials. Focus is on the alternative formulations suited to describe the wave propagation in the material, according to the classic canons of solid or structural mechanics. Adopting the centrosymmetric tetrachiral microstructure as prototypical periodic cell, the frequency dispersion spectrum resulting from a synthetic lagrangian beam-lattice formulation is compared with its counterpart derived from different continuous models (high-fidelity first-order heterogeneous and equivalent homogenized micropolar continuum). Asymptotic perturbation-based approximations and numerical spectral solutions are cross-validated. Adopting the low-frequency band gaps of the material band structures as functional targets, parametric analyses are carried out to highlight the descriptive limits of the synthetic models and to explore the enlarged parameter space described by high-fidelity models. The final tuning of the mechanical properties of the cellular microstructure is employed to successfully verify the wave filtering functionality of the tetrachiral material.

1. Introduction

Periodic materials are characterized by a repetitive microstructure realizing a regular pattern of elementary cells. The research interest in these materials is being currently renewed for their high mechanical performances and smart technological applications in the naval, aerospace, nuclear, sport, biomedical engineering fields. The key of such an exponential success can be recognized in their non-conventional, or even extreme mechanical properties and tunable multi-purpose functionalities [1,2].

Within the wide realm of microstructured periodic materials, two leading research lines can be identified. The first line pays attention to the homogenization or continualization in local and nonlocal continua in which the overall constitutive tensors are determined by means of standard or generalized macro-homogeneity conditions [3–10]. The second line focuses on the assessment and customization of the acoustic dispersion properties associated to the propagation of Bloch waves across the material, either in its original periodic microstructure [11–15] or in its equivalent homogenized form [8,9,16–19]. In this respect, the periodic materials with a chiral or antichiral microstructure of the elementary cell [20–22], consisting of stiff disks or rings, connected by a variable number of flexible ligaments, are particularly attractive for their potential as acoustic waveguides or phononic filters. In the current literature dealing with this material class, the pass and stop bands characterizing the band structures have been determined by solving the dispersion problem related to low-dimensional lagrangian models [23–28], high-fidelity micromechanical formulations accounting for the material heterogeneity at the microscale [11,12,14,29] and equivalent local and non-local homogenized continua [9,26–28]. The underlying idea is that, within certain physically admissible ranges, the geometric and mechanical parameters can be intended as freely tunable variables for customizing the acoustic dispersion properties of the material. To this purpose, resonant auxiliary oscillators (local resonators) can conveniently be introduced to realize acoustic metamaterials, featured by an enlarged configuration space of active degrees-of-freedom and a richer variety of tunable mechanical parameters [14,27,29–31]. Among the others, common customization criteria are the presence of selected harmonics in the band structure at a certain wavenumber [32,33], the opening of maximum-amplitude band gaps in the lowest possible frequency range [34–38], the maximal sensitivity of the spectrum to microstructural defects [39,40], the occurrence of negative refraction properties [41,42].

Asymptotic techniques may allow the multiparametric approximation of the direct and inverse dispersion problem for low-dimensional lagrangian models. Consequently, the conditions for the existence of pass and stop bands can be determined in a suited analytical – although

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approximate – form [40]. The relative optimization analyses may highlight how synthetic lagrangian models likely possess a low-dimensional parameter space, insufficient for the search of a satisfying solution for inverse spectral problems.

The present paper is devoted at exploring the dispersion properties of the tetrachiral material in the larger parameter space obtainable by removing some of the simplifying mechanical assumption limiting the simpler lagrangian model. Two alternative continuous models (high-fidelity first-order heterogeneous and equivalent homogenized micropolar continuum) are derived in parallel to the lagrangian beam-lattice formulation (Section 2). The frequency dispersion spectra resulting from all the models are compared to each other and cross-validated (Section 3). The qualitative and qualitative agreement between asymptotic perturbation-based approximations and numerical spectral solutions is discussed (Paragraph 3.1). Parametric analyses concerning the effects of variations in the enlarged space of geometric and mechanical parameters on the acoustic and optical surfaces are carried out (Section 4). Consequently, a satisfying tuning of the micromechanical properties is employed to successfully verify the filtering functionality of the material in the forced wave propagation (Paragraph 4.1). Concluding remarks are finally pointed out.

2. Tetrachiral material

2.1. Beam lattice model

The class of chiral and antichiral cellular materials is characterized by a periodic tessellation of the bidimensional plane. The elementary cell is strongly characterized by a microstructure composed by stiff circular rings connected by flexible straight ligaments, arranged according to different planar geometries including the trichiral, hexachiral, tetrachiral, anti-trichiral, antitetrachiral topologies [22]. Among the others, the tetrachiral material is featured by a monoatomic centrosymmetric cell in which the central stiff and massive ring (or disk) is connected to four tangent flexible and light ligaments (Fig. 1a). The periodic square cell has side length H. Each ring is mechanically modeled as a rigid annular body with mass M_r, rotational inertia J_r, mean radius R and transversal width t_r, (Fig. 1b). Each ligament is mechanically modeled as a linear unshearable beam, with material density ρ_b, transversal width t_b and natural length $L_b = H\cos\beta$, where the *chirality angle* $\beta = \arcsin(2R/H)$ is the ligament inclination angle with respect to the ideal line connecting the centers of adjacent rings. A linear elastic material, with Young's modulus E_b is assumed for all the beams.

The rigid body configuration is fully described by three planar *active* degrees-of-freedom, collected in the generalized displacement vector $\mathbf{q}_a = \mathbf{q}_1$ (Fig. 1c), referred to the internal node located at the ring barycenter. Due to the geometric periodicity, the cell boundary crosses the midspan of all the four ligaments. Consequently four external nodes are located at the midpoint of all the cell sides, each one possessing three planar *passive* degrees-of-freedom collected in the displacement vector $\mathbf{q}_p = (\mathbf{q}_2, \ldots, \mathbf{q}_5)$.

Assuming the ligaments rigidly connected to the ring, a lagrangian beam lattice model can be formulated. The free undamped vibrations of the lagrangian model are governed by a linear equation, defined in the full configuration vector $\mathbf{q} = (\mathbf{q}_a, \mathbf{q}_p)$

$$\begin{bmatrix} \mathbf{M} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix}\begin{pmatrix} \ddot{\mathbf{q}}_a \\ \ddot{\mathbf{q}}_p \end{pmatrix} + \begin{bmatrix} \mathbf{K}_{aa} & \mathbf{K}_{ap} \\ \mathbf{K}_{pa} & \mathbf{K}_{pp} \end{bmatrix}\begin{pmatrix} \mathbf{q}_a \\ \mathbf{q}_p \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{f}_p \end{pmatrix} \tag{1}$$

where dot indicates differentiation with respect to time and $\mathbf{O}$ stands for different-size empty matrices. Adopting a lumped mass description, the non-null mass submatrix $\mathbf{M}$ is diagonal. The symmetric submatrices $\mathbf{K}_{aa}$ and $\mathbf{K}_{pp}$ describe the stiffness of the active and passive nodes, respectively. The rectangular submatrix $\mathbf{K}_{ap} = \mathbf{K}_{pa}^T$ account for the elastic coupling among the active and passive nodes. The mass and stiffness matrices are reported in details in the Appendix. The vector $\mathbf{f}_p$ collects the reactive forces exerted by the adjacent cells on the passive nodes. The passive displacement and force vectors can be ordered and partitioned as $\mathbf{q}_p = (\mathbf{q}_p^-, \mathbf{q}_p^+)$, $\mathbf{f}_p = (\mathbf{f}_p^-, \mathbf{f}_p^+)$ to separate the variables $(\mathbf{q}_p^-, \mathbf{f}_p^-)$, related to the left/bottom sides of the cell boundary (composed by the external nodes 2, 3 shown in Fig. 1c), from the variables $(\mathbf{q}_p^+, \mathbf{f}_p^+)$ related to the right/top sides (composed by the external nodes 4, 5). According to this decomposition, the dynamic (upper) part of the Eq. (1) can be written

$$\mathbf{M}\ddot{\mathbf{q}}_a + \mathbf{K}_{aa}\mathbf{q}_a + \mathbf{K}_{ap}^+\mathbf{q}_p^+ + \mathbf{K}_{ap}^-\mathbf{q}_p^- = \mathbf{0} \tag{2}$$

whereas the quasi-static (lower) part can be written

$$\begin{bmatrix} \mathbf{K}_{pa}^- \\ \mathbf{K}_{pa}^+ \end{bmatrix}\mathbf{q}_a + \begin{bmatrix} \mathbf{K}_{pp}^= & \mathbf{K}_{pp}^\mp \\ \mathbf{K}_{pp}^\pm & \mathbf{K}_{pp}^\# \end{bmatrix}\begin{pmatrix} \mathbf{q}_p^- \\ \mathbf{q}_p^+ \end{pmatrix} = \begin{pmatrix} \mathbf{f}_p^- \\ \mathbf{f}_p^+ \end{pmatrix} \tag{3}$$

According to the Floquet-Block theory for two dimensional discrete model [43], the quasi-periodic conditions governing the propagation of planar wave can be imposed on the passive displacement/forces at the cell boundary, requiring

$$\mathbf{q}_p^+ = \mathbf{L}_k\mathbf{q}_p^-, \qquad \mathbf{f}_p^+ = \mathbf{L}_k\mathbf{f}_p^- \tag{4}$$

where $\mathbf{L}_k$ is a square *transfer* matrix that can be expressed in the diagonal block form

$$\mathbf{L}_k = \mathrm{diag}(e^{ik_1H}\mathbf{I}, e^{ik_2H}\mathbf{I}) \tag{5}$$

where $\mathbf{I}$ is the 3-by-3 identity matrix, while k_1 and k_2 are the two components of the wavevector $\mathbf{k} = (k_1, k_2)$, that is, the wavenumbers of the horizontally and vertically propagating waves, respectively.

The conditions (4) can be introduced in the quasi-static Eq. (3) to reduce the number of independent passive displacements [25]. Therefore, the linear quasi-static laws

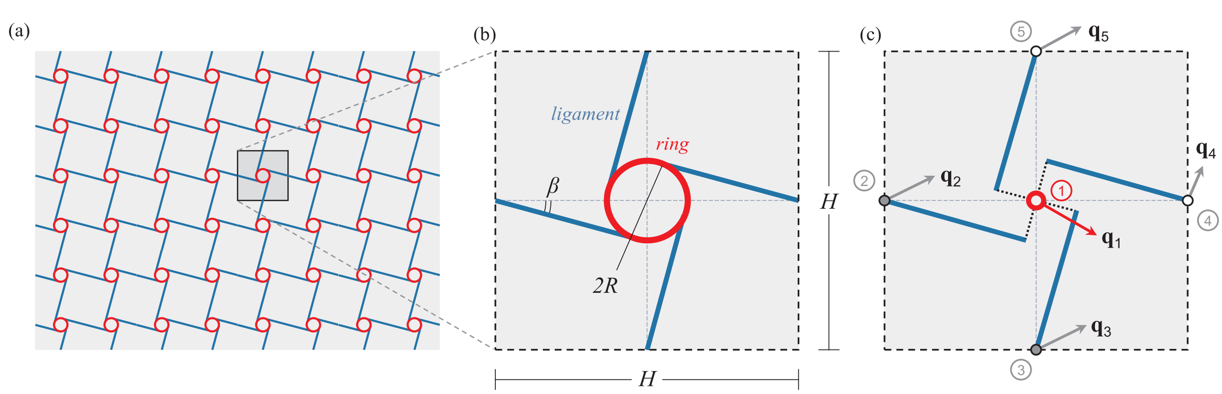

Fig. 1. Tetrachiral metamaterial (a) repetitive planar pattern, (b) periodic cell, (c) beam lattice model.