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Meshless analysis of metallic and composite beam structures by advanced hierarchical models with layer-wise capabilities

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ABSTRACT

This paper proposes radial basis function (RBF)-based meshless solutions for static and dynamic analyses of metallic and laminated beam-like structures with various boundary conditions. Making use of the Carrera Unified Formulation (CUF), the three-dimensional displacement field can be reduced to one-dimensional displacements related to the axial direction multiplied by cross-sectional kinematics. Locally supported Wendland's C^6 RBF is employed to interpolate the displacements in the axial domain and Hierarchical Legendre Expansion (HLE) is used to expand the kinematic unknowns over the cross-section domain, being endowed with Layer-Wise (LW) ability. The principle of virtual displacements (PVD) is adopted to derive the governing equation in a strong form, accompanied by the advent of fundamental nuclei, which are independent of the transverse assumptions in the cross section domain. Different numerical assessments on metallic and laminated structures are addressed to show the performance of RBF-based HLE models in terms of displacements, stresses and vibration modes. 3D accuracy of the obtained results is demonstrated by comparison with 3D FEM solutions provided by well-known commercial softwares (Ansys and Abaqus).

1. Introduction

Beams, as primary and secondary components, can be found in many diverse engineering fields, such as aeronautical and aerospace, mechanical, civil and ocean industries. During service lives, these structures are subjected to various loading conditions, e.g. cyclic loads, mechanical vibrations and impulsive loads. A better understanding of resultant static and dynamic responses is of paramount importance for the safe and optimal structural design. In theory, the accurate simulation of these behaviours requires burdensome three-dimensional finite element method (FEM). In order to resolve the issues related to high computing costs radically, dimensional reduction models with enhanced analytical abilities are preferred and constantly improving for the past several years. A brief, though not exhaustive, review is given hereafter.

Classical 1D beam theories, known as Euler-Bernoulli (EBBM) and Timoshenko (TBM) theories, has been widely adopted for the analysis of slender homogeneous structures with bending-dominated deformation and a large number of fruitful results have been contributed by considerable papers [1–4]. However, refined 1D models should be built to capture more non-classical phenomena, such as warping, torsion and coupling modes, which arise when beams with lower shear moduli in comparison with Young's moduli or with complex loading conditions are considered. A possible scheme of advanced beam theories built by different techniques can be summarized as follows: (i) insertion of warping function; (ii) the Saint-Venant solution; (iii) the Variational Asymptotic Method (VAM); (iv) the Generalized Beam Theory (GBT); (v) Higher-order shear deformation models. A thorough review of recent developments on refined beam theories is stated by Carrera et al. [5]. Vlasov [6] initially developed the first version of thin-walled beam theory, where the displacement field was improved via the introduction of an extra warping function. Unfortunately, this assumption is only applicable to the case of thin-walled beams with open cross-sections subjected to the uniform torsion, bringing about the neglect of in-plane deformations and the shear deformation of the middle surface [7]. Ferrero et al. [8] presented an analytical method to study the torsional behaviour of thin-walled composite beams with midplane symmetric under a twisting moment. This method can be extended to beams with closed section and also constrained warping effect can be taken into account, as well. Sapountzakis and his colleagues [9–11] investigated non-uniform torsion of homogeneous and composite bars of arbitrary variable cross-sections employing the boundary element method, in

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cases where the primary and second warping functions were introduced. Starting from 3D Saint-Venant solution, El Fatmi [12,13] and Ladeveze and Simmonds [14] presented a non-uniform warping beam theory including three warping functions corresponding to torsion and shear forces. GBT was first introduced by Schardt [15] for thin-walled structures, in which the displacement of the mid-wall cross-section can be described using a piece-wise function. The extension of GBT to composite beams can be referred to [16]. VAM, introduced by Berdichevskii [17] employs the series expansion of the characteristic parameter, such as the thickness of the cross section h, to obtain the beam theory with known accuracy. As a viable alternative approach to refined beam models, higher-order shear deformation models were introduced to provide different distributions of the transverse shear strains along the thickness. Ghugal [18] developed a trigonometric shear deformation for the flexure and vibration analyses of thick isotropic beams. Their model used the sinusoidal function in the displacement field to represent the shear deformation effects, satisfying the traction free boundary conditions at the top and bottom surfaces of the beam. Analogously, hyperbolic, and exponential shear deformation theories have been presented by various authors [19,20].

Beams composed of composite materials deserve special attention due to their high strength and stiffness to weight ratios. They are in general anisotropic and present discontinuous mechanical properties in the layer thickness direction. So called zig-zag (zz) forms of displacement fields, i.e., piece-wise distribution along the laminate thickness direction as well as interlaminar continuity (IC) of transverse both shear and out of plane components need to be addressed. A number of review papers are available, see Kapania and Raciti [21] and Carrera [22,23]. These theories can be classified into two categories: Equivalent Single Layer (ESL) and Layer-Wise theories, depending on the relation between computational costs and the number of layers. A short review of the contribution which is useful for our propose is given below. Khedeir and Reddy [24] developed analytical solutions of various refined beam theories to study free vibration behaviour of cross-ply laminated beams through ESL and concluded that EBBM gave less accurate results than other theories. Kant and Manjunath [25] introduced a non-linear variation of longitudinal displacements through the thickness via Taylor Expansion (TE) and the results were compared with those from an earlier investigation in terms of displacements, stresses and modes [26]. Karama et al. [27] proposed a new exponential shear deformation theory in conjunction with the Heaviside step function to predict the mechanical behaviour of multi-layered laminated composite beams. The authors concluded that the property of exponential function was better than sine and cosine functions and its LW ability improved the accuracy of the transverse shear stress at layer interfaces. Shimpi and Ghugal [28] presented a LW trigonometric shear deformation theory for the analysis of two-layered cross-ply laminated beams. It was pointed out that the number of primary variables was even less than that of TBM. The extension of [28] to general lamination can be found in [29], which used the Lagrangian linear interpolation functions to achieve LW ability. In addition, various researchers employed zig-zag function capable to capture the zig-zag phenomena in the displacement field along the thickness [30,31].

Carrera et al. [32] introduced a unified beam theory, known as Carrera Unified Formulation (CUF) for the mechanical analyses of various types of beams. In the light of CUF, the displacement field can be decomposed into the product of the cross section function and the generalized displacement function related to the axial coordinate. Four different expansions, i.e., Taylor Expansion (TE), Hierarchical Legendre Expansion (HLE), Lagrange Expansion (LE) and Chebyshev Expansions (CE) are usually adopted to characterise the cross section function. With regard to CUF-TE, Carrera et al. [32] used a 1D beam element to approximate the generalized displacement function, being capable of furnishing 3-D stress states of beams with solid and thin-walled cross sections. The extension of the aforementioned isotropic and static problems to anisotropic (composite material) case and free vibration problems accomplished in the weak form can be found in literature [33,34]. Moreover, the strength of the CUF- TE model solved by strong form, i.e., radial basis function (RBF) and dynamic stiffness method (DSM) concerning free vibration and thermal-mechanical analyses of isotropic and composite beams can be referred to Pagani et al. [35,36] and Giunta et al. [37]. In CUF-HLE, a set of Legendre functions were defined in the natural coordinate system and mapped into the physical coordinate system, obtaining both ESL and LW solutions. FEM application of CUF-HLE to isotropic and laminated structures can be seen in [38,39].

The application of other CUF models can be seen in [40–42]. The present work focuses its attention on the development of a LW beam theory that makes use of the CUF-HLE model in combination with RBF solution. RBF method is a truly meshless method, first introduced by Kansa [43] to solve the partial differential equations. Compared with FEM, this method only places a cluster of nodes on the domain and boundary with no need of mesh and its interpolation function is distance dependent, thus insensitive to the dimension. In recent years, it has become an outstanding candidate for the solution of the partial differential equations due to the merit of superior exponential convergence, easy implementation and high accuracy. Subsequently, its extensive use in the application of elastic problems (mechanical behaviours of beam, plate and shell) can be found in [44,45].

The rest of this paper is structured as follows: preliminary knowledge on the anisotropic elasticity, CUF and HLE is introduced in Section 2; then, the strong form governing equation for static and free vibration problems are derived in Section 3; the RBF method and its formulation into CUF-HLE model are subsequently discussed in Section 4 and Section 5; numerical assessments on mechanical behaviours of metallic and composite beams are carried out in Section 6. Finally, the main conclusions and guidelines are remarked in Section 7.

2. Unified formulation of beam theories

2.1. Preliminary

Consider a generic 3D beam structure in the Cartesian coordinate system with cross section Ω lying on the x–z plane and length L coinciding with y-axis, as shown in Fig. 1. The 3D displacement field $\mathbf{u}(x, y, z)$, strain field $\boldsymbol{\epsilon}$ and stress field $\boldsymbol{\sigma}$ can be written as:

$$
\begin{aligned}
\mathbf{u}(x, y, z, t) &= \{u_x \ u_y \ u_z\}^{\mathrm{T}} \\
\boldsymbol{\epsilon} &= \{\epsilon_{yy} \ \epsilon_{xx} \ \epsilon_{zz} \ \epsilon_{xz} \ \epsilon_{yz} \ \epsilon_{xy}\}^{\mathrm{T}} \\
\boldsymbol{\sigma} &= \{\sigma_{yy} \ \sigma_{xx} \ \sigma_{zz} \ \sigma_{xz} \ \sigma_{yz} \ \sigma_{xy}\}^{\mathrm{T}}
\end{aligned} \tag{1}
$$

where index T is the transpose operator. The geometric linear relation

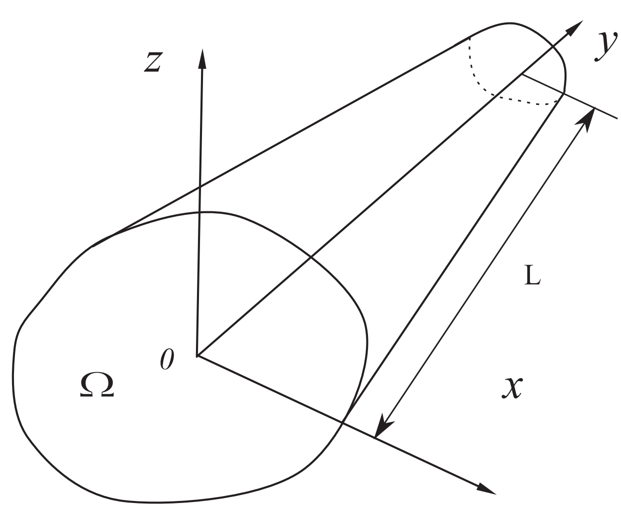

Fig. 1. Coordinate system for the beam structure.