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Static, dynamic and buckling analyses of 3D FGM plates and shells via an isogeometric-meshfree coupling approach

Pengfei Tan[a], Nhon Nguyen-Thanh[a,*], Timon Rabczuk[b], Kun Zhou[a,*]

[a] School of Mechanical and Aerospace Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore
[b] Institute of Structural Mechanics, Bauhaus-University Weimar, Marienstr. 15, D-99423 Weimar, Germany

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ABSTRACT

This paper develops a three-dimensional (3D) isogeometric analysis (IGA) and meshfree coupling approach to investigate the static, dynamic and buckling behaviors for plates and shells of functionally graded material (FGM). The meshfree method and IGA are coupled using the higher-order consistency condition in the physical domain so that the higher-order continuity of basis functions is guaranteed, and the topological complexity of the global volumetric parameterization for IGA to build the 3D geometry can be overcome. By employing IGA elements on the domain boundary and meshfree nodes in the interior domain, the approach preserves the advantages of the exact geometry and flexible discretization in the problem domain. Based on the coupling approach, the analyses for FGM plates and shells are carried out, and the effects of the material volume fraction, the side-to-thickness ratio and the curvature of the cylindrical shell on the deflection, natural frequency, and buckling load are investigated. The coupling approach is verified by comparing with the solutions obtained from other existing theories.

1. Introduction

Functionally graded material (FGM) is a type of composite material which possesses smoothly and continuously variable material properties in the thickness direction. The novel combination of material ingredients endows FGM with excellent mechanical performance characteristics such as high strength-to-weight ratio, thermal and corrosion resistance, and fatigue strength. Among FGM structures, plates and shells occupy primary roles in engineering applications such as automobiles, aircrafts, nuclear power plants and medical apparatus.

As FGM plates and shells are increasingly applied to industrial fields, various plate and shell theories have been proposed for the structural analyses. Based on hypotheses for the shear deformation distribution, the 3D geometry can be simplified as a two-dimensional (2D) model for analysis. The 2D theory can be classified into three categories: the classic theory [1], first-order shear deformation theory (FSDT) [2,3] and higher-order shear deformation theory (HSDT) [4–8]. The classical theory that follows Kirchhoff-Love assumptions is only suitable for thin plates and shells as it neglects the effects of the shear deformation. The FSDT considers the linearly distributed transverse shear deformation, while it may lead to the shear locking and non-zero shear stress boundary condition. The HSDT circumvents the disadvantages of FSDT and obtains more accurate solutions by incorporating high-order terms to approximate the displacement field. Compared with the 2D theories, the 3D analysis models do not involve simplifications and assumptions that may give rise to inaccurate solutions [9]. The 3D theories not only achieve more reliable solutions but also enable clearer physical insights [10]. Furthermore, the 3D models can provide a full frequency spectrum for the dynamic analysis of FGM structures [11,12]. The 3D static, dynamic and buckling analyses for FGM structures have been conducted by numerous researchers [13–16].

To solve the 2D or 3D analysis formulations for FGM plates and shells, a variety of computational methods have been developed, including analytical solutions and numerical methods such as the finite element method (FEM), meshfree method and isogeometric analysis (IGA). The analytical solutions for the static and dynamic analyses of FGM plates and shells can be found in [12,15,17–19]. The conventional FEM is a powerful tool for the FGM structural analyses [20–24]. However, FEM still suffers from limitations such as mesh distortion at large deformations, intensive remeshing requirements [25] and only C^0 continuity between elements, which can be partially overcome by the meshfree method and IGA.

The meshfree method employs a set of arbitrarily scattered nodes to discretize the problem domain without connected elements. Compared with FEM, the meshfree method can obtain a more accurate approximation for complex structures, higher-continuity basis functions, and

* Corresponding authors.
E-mail addresses: thanhnhon@ntu.edu.sg (N. Nguyen-Thanh), kzhou@ntu.edu.sg (K. Zhou).

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flexible local refinement. Among a variety of meshfree methods [26–29], the reproducing kernel particle method (RKPM) [30] is one of representative meshfree methods and has been utilized to solve solid mechanics problems [31,32]. The applications of meshfree methods for the analyses of plates and shells are reported in [33–38]. Additionally, since the Kronecker delta condition is not satisfied in the meshfree method which results in the difficulty to apply the essential boundary condition, several improved meshfree methods have been proposed and employed in the plate analyses [39–45].

IGA proposed by Hughes et al. [46], integrating computer aided design and FEM, has attracted great attentions because of the exact geometry representation, higher-order continuity, flexible k-refinement, and robustness for the large deformation. IGA has been extensively applied for structural analyses [47–51], fracture mechanics [52–54] and fluid mechanics [55]. IGA maintains the exact geometry by using the non-uniform rational B-spline (NURBS) as basis functions to create the geometric model, which is a great advantage for analysing shells and complex structures [56,57]. In addition, the arbitrary continuity order of NURBS basis functions can be controlled, which is needed for the HSDT [58–61].

To exploit the advantages of the meshfree method and IGA, the coupling of the two methods has been developed recently. Wang et al. [62] proposed a coupling of the B-spline basis functions and meshfree shape functions using the reproducing conditions. Since it is defined in the parametric domain, the global geometry parameterization is required. Rosolen et al. [63] combined the local maximum entropy meshfree method and IGA in the physical domain, which concisely addresses the volume discretization and flexible local refinement simply. However, the coupling approach only satisfies first-order continuity. Considering the challenges in the two approaches, Valizadeh et al. [64] developed an IGA-meshfree coupling approach using higher-order consistency conditions in the physical domain, which preserves the arbitrary approximation order of the coupling basis functions and avoids the complexity of a global parameterization to build the 3D problem domain. Therefore, in this work, the IGA-meshfree coupling approach is used to develop 3D analysis formulations for FGM plates and shells.

In this paper, a novel 3D IGA-meshfree coupling approach is developed to analyze FGM plates and shells. IGA is implemented for the exact description of the geometric model on the domain boundary, while the interior domain is discretized by meshfree nodes. This coupling approach based on the higher-order consistency conditions is established in the physical domain, which has higher-continuity basis functions and alleviates the difficulty to construct 3D complex geometry in the global parametric domain for IGA [64]. The effects of material ingredients, boundary conditions, the plate thickness and the curvature of the cylindrical shell on the deflection, natural frequency and buckling load are investigated. Numerical examples are presented to demonstrate the efficiency and accuracy of the coupling approach.

This paper is outlined as follows. The following section introduces the coupling of the meshfree method and IGA. In Section three, 3D formulations for FGM plates and shells analyses using the IGA-meshfree coupling approach are presented. Several numerical examples for FGM plates and shells are given in Section four. Finally, conclusions are drawn in Section five.

2. IGA-meshfree coupling approach

The basis functions for the NURBS-based IGA and RKPM are provided in this section. The IGA and RKPM are coupled in a narrow boundary region using the consistency conditions.

2.1. NURBS basis functions

NURBS basis functions are built on B-spline basis functions by the projective transformation. In the parametric coordinate, B-spline basis functions are expressed by a non-decreasing set of knot values called knot vector $\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$ ($\xi_i \in \mathbb{R}$), where p is the polynomial order and n is the number of basis functions. Starting from the order $p = 0$, the recursive form of the B-spline basis functions is defined as [46]:

$$N_{i,0} = \begin{cases} 1 & \text{if} \xi_i \leqslant \xi \leqslant \xi_{i+1}, \\ 0 & \text{otherwise} \end{cases} \quad \text{for } p = 0 \tag{1}$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi), \quad \text{for } p > 0., \tag{2}$$

As three sets of knot vectors $\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$, $\mathrm{H} = \{\eta_1, \eta_2, \ldots, \eta_{m+q+1}\}$ and $\Pi = \{\zeta_1, \zeta_2, \ldots, \zeta_{n+p+1}\}$ are given in the ξ, η and ζ directions, respectively, 3D NURBS basis functions are obtained by using the tensor product:

$$R_{i,j,k}(\xi,\eta,\zeta) = \frac{N_{i,p}(\xi) M_{j,q}(\eta) L_{k,r}(\zeta) w_{i,j,k}}{\sum_{\hat{k}=1}^{l} \sum_{\hat{j}=1}^{n} \sum_{\hat{i}=1}^{m} N_{\hat{i},p}(\xi) M_{\hat{j},q}(\eta) L_{\hat{k},r}(\zeta) w_{\hat{i},\hat{j},\hat{k}}}, \tag{3}$$

where $w_{i,j,k}$ is the weight, and $N_{i,p}$, $M_{i,q}$ and $L_{k,r}$ are the basis functions. The NURBS solid is expressed as

$$V(\xi,\eta,\zeta) = \sum_{\hat{k}=1}^{l} \sum_{\hat{j}=1}^{n} \sum_{\hat{i}=1}^{m} R_{i,j,k}(\xi,\eta,\zeta) P_{i,j,k}. \tag{4}$$

where m, n and l are the numbers of basis functions in the ξ, η and ζ directions, respectively, and $P_{i,j,k}$ is the control point.

2.2. RKPM basis functions

In the physical problem domain discretized by a set of particles $\{\mathbf{x_i}\}_{i=1}^{N_\mathrm{P}}$, where N_P is the number of meshfree particles, the RKPM basis functions [31] are defined as

$$\psi_i(\mathbf{x}) = C(\mathbf{x}, \mathbf{x} - \mathbf{x}_i) \phi_a(\mathbf{x} - \mathbf{x}_i), \tag{5}$$

where ϕ_a is the kernel function [65]:

$$\phi_a(s) = \begin{cases} 1 - 6s^2 + 8s^3 - 3s^4 & \text{for } s \leqslant 1 \\ 0 & \text{for } s > 1 \end{cases}, \quad s = \frac{\|\mathbf{x} - \mathbf{x}_i\|}{r_0}, \tag{6}$$

where r_0 is the radius of spherical support domain for the 3D geometry, which is calculated as follows:

$$r_0 = \lambda d, \; d = \max(|x - x_i|, |y - y_i|, |z - z_i|) \tag{7}$$

where λ is the scaling factor taken as 2.5 and d is the maximum distance of adjacent nodes in three axial directions.

The correction function $C(\mathbf{x}, \mathbf{x} - \mathbf{x}_i)$ is expressed as a linear combination of polynomial basis functions:

$$C(\mathbf{x}, \mathbf{x} - \mathbf{x}_i) = \mathbf{H}^\mathrm{T}(\mathbf{x} - \mathbf{x}_i) \mathbf{b}(\mathbf{x}), \tag{8}$$

where the quadratic polynomial is used in this work:

$$\mathbf{H}(\mathbf{x} - \mathbf{x}_i) = [1 (x - x_i)(y - y_i)(z - z_i)(x - x_i)(y - y_i)(y - y_i)(z - z_i)(z - z_i)(x - x_i)(x - x_i)^2 (y - y_i)^2 (z - z_i)^2]^\mathrm{T} \tag{9}$$

and the unknown coefficient $\mathbf{b}(\mathbf{x})$ is determined by imposing the pth order reproducing conditions as follows:

$$\sum_{i=1}^{N_\mathrm{P}} \psi_i(\mathbf{x}) \mathbf{H}(\mathbf{x} - \mathbf{x}_i) = \mathbf{H}(0). \tag{10}$$

The substitution of Eq. (5) into Eq. (10) results in:

$$\mathbf{M}(\mathbf{x}) \mathbf{b}(\mathbf{x}) = \mathbf{H}(0), \tag{11}$$

where the moment matrix $\mathbf{M}(\mathbf{x})$ are given as

$$\mathbf{M}(\mathbf{x}) = \sum_{i=1}^{N_\mathrm{P}} \mathbf{H}^\mathrm{T}(\mathbf{x} - \mathbf{x}_i) \phi_a(\mathbf{x} - \mathbf{x}_i) \mathbf{H}(\mathbf{x} - \mathbf{x}_i). \tag{12}$$