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Methodology for the analysis of post-tensioned structures using a constitutive serial-parallel rule of mixtures

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ABSTRACT

The main purpose of this paper is to develop a reliable method based on a three-dimensional (3D) finite-element (FE) model to simulate the constitutive behaviour of reinforced concrete structures strengthened with post-tensioned tendons taking into account the reduction of the pre-stressing stress due to the steel relaxation. The post-tensioned concrete is modelled as a composite material whose behaviour is described with the serial-parallel rule of mixtures (S/P RoM) (Rastellini et al, 2008; Martinez et al., 2008, 2014) whereas the stress relaxation of the steel is simulated using a viscoelastic model called Generalized Maxwell. A 3D FE model was used, where the nonlinear material behaviour and geometrical analysis based on incremental–iterative load methods were adopted. Validation by comparison with the analytic solution will be done for the case of a concrete beam with a linear steel tendon and for a parabolic pre-tensioned steel tendon embedded. Some viscoelastic cases are presented in order to perceive the behaviour of the Generalized Maxwell model. Several examples are shown where the capabilities of the method on large scale structures are exhibited.

1. Introduction and state of the art

One of the most commonly used methods to simulate the post-tensioned system of reinforced concrete structures (for straight and parabolic shaped tendons) consists in adding concentrated loads at the anchoring zones and an ascending distributed load that represents the effect of the curvature of the tendon along its path [4]. This method has the advantage that it is really easy to apply but is limited to simple geometries in which the equivalent system of forces can be estimated properly. Another more sophisticated method consists in a combination of the finite element method (that is used to model the concrete) and bi-articulated elements connecting pair of nodes of the FE mesh in order to represent the steel tendon [5–7]. Many attempts have been made to correctly model the behaviour of pretensioned concrete members using this second technique.

Arab et al. [8] compared several modelling techniques concerning the pre-stressed tendons. One model uses an extrusion technique; a second model is built following the concept of embedded reinforcement in which the pre-stressed tendons are implemented using one-dimensional elements which are embedded in the concrete continuum elements. For both models, bondslip behaviour is achieved by implementing the frictional nature at the concrete-strand interface. However, in the extruded model, this is achieved through contact surface algorithms while the embedded model uses nodal constraints and master–slave connections It is concluded that a correct assessment of the overall behaviour of the pre-tensioned elements can be achieved with both techniques

As stated by Kwan et al. [7], a nonlinear finite element model based on a commercial code (ANSYS) was used by Kaewunruen and Remennikov [9] to analyse railway prestressed concrete sleepers. This model included brick elements to represent the concrete matrix with embedded three-dimensional truss elements simulating the prestressing.

Stephen [10] used a comprehensive three-dimensional finite element model developed with the commercial code (ABAQUS) to simulate the long-term behaviour of precast/prestressed concrete bridges. This model included elastoplastic material modelling capable of capturing the nonlinear behaviour of various concrete members (e.g., deck slab, pretensioned concrete girders) due to long term effects such as creep and shrinkage. The model required an external subroutine to facilitate specific operations such as prestressing and application of the long term effects.

Rabczuk and Eibl [11] proposed a coupled element free Galerkin method to analyse prestressed concrete beams under quasi-static loading. The constitutive law governing the concrete medium was based on a coupled damage-plasticity model. The reinforcement was

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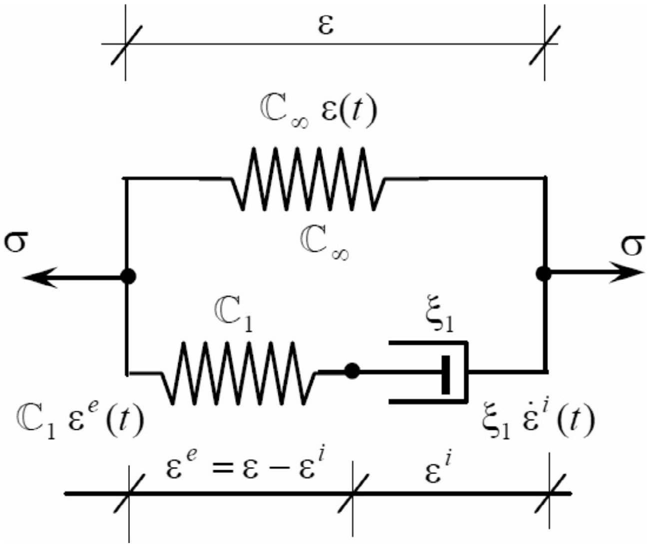

Fig. 1. The generalized Maxwell Model.

modelled as discrete beam elements so that the interaction between concrete and reinforcement can be modelled. The bond model included two modes of failure: pullout failure, and splitting failure. The formulation of the bond model was based on the radial stress–radial strain relation with three distinct domains: the nonlinear material behaviour including the initiation and propagation of cracks, linear softening, and residual strength.

More recently, Ayoub and Filippou proposed a nonlinear model for simulation of the pretensioned prestressed concrete girders [12]. The modelling approach consisted of three main components: concrete girder simulated as a beam–column, prestressing strands modelled as truss elements, and a bond element to model the prestress transfer between the concrete and strands. The constitutive laws governing the nonlinear response of concrete and strands were based on discretization of the media into fibers with uniaxial hysteric models. The bond model at the interface between the concrete and strand was formulated using special bond stress–slip relations. The pretensioning mechanism was divided into discrete time steps representing various stages of the operation.

Another field where prestressing techniques are used is in the study of nuclear power plant buildings. For instance, Hu and Lin [13] conducted an analytical study also using ABAQUS finite element program to predict the ultimate pressure capacity of the PWR (Pressure Water Reactor) prestressed concrete containment at Maanshan nuclear power

Table 1
Algorithm for obtaining the stress of the generalized Maxwell model.

1. Obtaining the strain

$$[\boldsymbol{\varepsilon_{ij}}]^{\boldsymbol{t+t}}$$

2. Stress integration

$$[\sigma_{ij}]^{t+\Delta t} = [\sigma_{ij}]^{t}\cdot \mathrm{e}^{-(\Delta t)/r_1} - \mathbb{C}_{ijkl}[\varepsilon_{kl}]^{t}\cdot \mathrm{e}^{-(\Delta t)/r_1}\left[1+\frac{\mathbb{C}_1}{\mathbb{C}_0\xi}\frac{\Delta t}{2}\right] + \mathbb{C}_{ijkl}[\varepsilon_{kl}]^{t+\Delta t}\cdot\left[1-\frac{\mathbb{C}_1}{\mathbb{C}_0\xi}\frac{\Delta t}{2}\right]$$

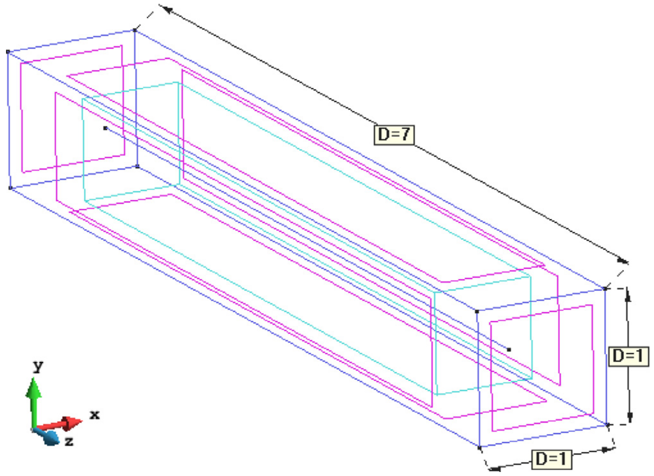

Fig. 3. Geometry of the beam and the steel tendon (units in m).

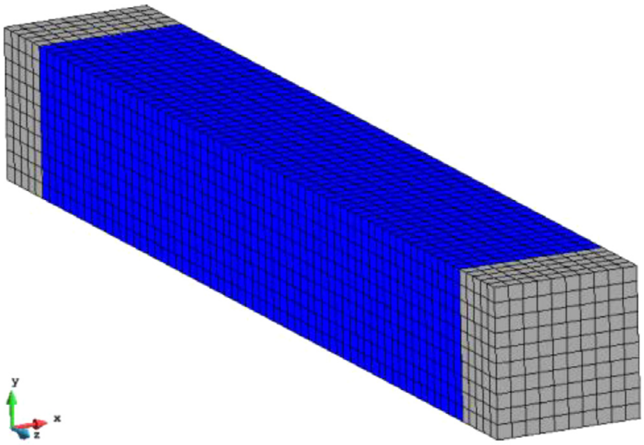

Fig. 4. Finite element mesh with 7000 linear hexahedra.

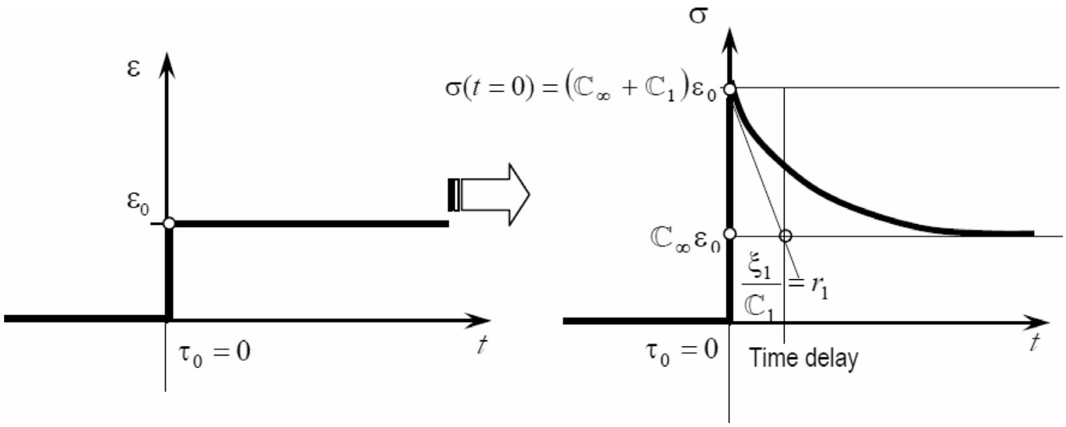

Fig. 2. Response of the generalized Maxwell Model under constant deformation.