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Exact static analysis of eccentrically stiffened plates with partial composite action

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ABSTRACT

Stiffened plates are common forms used in engineering structures. Extensive research efforts have been devoted to investigating the behavior of stiffened plates under static loading. In the existing models, two issues have not been always appropriately addressed. One is the eccentricity of the stiffeners and the other is the composite action between the plate and stiffeners. In the present paper, static deformation of an eccentrically stiffened plates with partial composite action was analyzed by using the variational approach. In order to investigate the eccentricity and composite action issues, the stiffened plates were idealized as assemblies of plate and beam elements, in such way the stiffeners are discretely connected to the plate elements. Based on the principle of the minimum potential energy, the governing differential equations were derived by using the variational approach with taking into consideration of strain energy of connectors between plate and stiffeners and associated boundary conditions as well. Five displacement functions were defined as double Fourier series to solve the static deflection of simply-supported eccentrically stiffened plates. A relative stiffness factor K with a range of zero to one was introduced to represent the composite action between the plate and stiffeners from no interaction (i.e., $K = 0$) to rigidly connected (i.e., $K = 1$). Finally, the proposed energy approach was compared with previously published methods through numerical examples of the plates reinforced by one to fourteen stiffeners in two orthogonal directions.

1. Introduction

Structural plate systems reinforced by eccentric stiffeners in one or two orthogonal directions have been extensively used in a large variety of engineering structures. Reinforcing of the plate with stiffeners can increase the load carrying capacity and prevent buckling thus to improve the plate's structural efficiency (e.g., strength-to-weight ratio). In civil engineering, for instance, the bridge decks generally consist of plates reinforced with rectangular, triangular or trapezoidal stiffeners, and composite floor systems in building structures are primarily constructed with floor decks stiffened by longitudinal joists and transverse elements. Moreover, the application of stiffened plates is especially indispensable in aerospace and marine structures, where weight minimization of the components is of paramount significance. Stiffened plates are generally employed in the hull, deck, bottom and superstructure of a ship. Similarly, the aircraft wings and fuselage consist of skin with an array of stiffening ribs [1]. The widespread applications of stiffened plates have led to considerable studies on their structural properties and performance. In fact, the behavior of stiffened plates under static loading has been studied for more than one century [2–6].

An effective analytical procedure is essential to the optimum design of stiffened plates. However, the static analysis of stiffened plates does not lend itself to easy analytical solutions even for simple geometries and boundary conditions. One quite popular method idealizes a stiffened plate as an equivalent orthotropic plate (i.e., orthotropic plate analogy) with uniform thickness by using "smeared out" technique on accounting for their resemblance with plates of truly orthotropic materials. A common premise of such technique is that the stiffeners are closely spaced and their rigidities do not dominate the plate rigidity [7]. Otherwise, significantly erroneous results may be produced. Instead of smearing the stiffeners, another approach is to consider them as discrete elements, represented by the Dirac delta functions [8,9]. In such approach, the Dirac delta function representation of the stiffeners is satisfactory as long as the width of the stiffeners is not comparable to the stiffener spacing. Moreover, approximate methods using energy principles [10] as well as numerical methods have been employed for the analysis of stiffened plates such as finite difference method [11], finite element method (FEM) [12–15], finite strip method [16,17], boundary element method(BEM) [18–21], differential quadrature method (DQM) [22] and sequential quadratic programming (SQP) [23].

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In addition, based on the fact that the finite integral transform method has been successfully developed in recent years for flexural analysis of plates with complex boundary conditions [24–27], it is expected that the method can also be applied for stiffened plates with complicated boundaries.

In the employed models for the development of aforementioned methods, two aspects have not been always addressed adequately. One is the eccentricity of the stiffeners and the other is the composite action between the plate and stiffeners. Most idealized orthotropic plate models assume that the stiffeners are symmetrical arranged above and below the plate. Such assumption yields to fairly simple and convenient design methods [2–4,28,10,1]. Furthermore, although the interaction between the stiffeners and plate has been considered as reported in some literature [22,29], stiffeners are generally assumed to be rigidly attached to the plate. However, such treatment is not always appropriate in real-world applications [8]. In practice, the stiffeners are often spot-welded or discretely fastened to a plate by using screws or bolts. Full composite action is achieved by rigid connectors which are able to prevent slip to arise in the interface between the plate and stiffeners, otherwise partial composite action will be resulted in. Thus, the partial composite action needs to be accounted for analysis of composite stiffened plates.

Furthermore, over last several decades, the partial composite action has been extensively investigated for composite beams such as steel-concrete beams [30–34]. Few research has been devoted to such issue of stiffened plates. Although the composite T-beam formed by a floor slab, a joist and shear connectors between them is a simplified model for composite floors and has been effectively applied for design of steel-concrete composite structures in practice, a refined plate model such as stiffened plates with partial composite action would be desirable for analysis and design of two-way floors. Initialed from the year of 2000, long-term efforts have been made by Sapountzakis and his colleagues to develop comprehensive models to investigate the forces and deformations at the interface of the plate and stiffeners [29,35–40]. In their models, the stiffeners were isolated from the plate through sections parallel to the lower outer surface of the plate, and various interface forces and actions were investigated by using the Analog Equation Method, a BEM based technique. Such models were originally proposed to solve the bending and vibration problems of composite steel-concrete structures with deformable connection. Very few applications have been found for other stiffened plate structures such as lightweight timber or steel floor systems. This would be partly because such theoretical treatments in applied mechanics involving BEM-based techniques are far beyond the technical abilities of normal researchers in Structural Engineering. An alternative straightforward and effective method would be more appealing.

In the present study, an analytical method with the exact theoretical solution was developed for static analysis of eccentrically stiffened plates with partial composite action based on the variational method. Both the aforementioned eccentricity and composite action issues were considered by idealizing the stiffened plates as assemblies of plate and beam elements. The study was motivated by a research on a practical design requirement of calculating the maximum deflection of lightweight floors resulted from 1 kN concentrated load to evaluate vibration performance. Based on the principle of the minimum potential energy, the governing differential equations are first derived by using variational approach with taking into consideration of partial composite action between plate and stiffeners, followed by associated boundary conditions. Then, five displacement functions were defined as double Fourier series to solve the static deflection of simply-supported stiffened plates. A relative stiffness factor K with a range of zero to one was introduced to represent the composite action between the plate and stiffeners from no interaction (i.e., $K = 0$) to full composite action (i.e., $K = 1$). Finally, the proposed energy approach was compared with previously published methods by numerical examples of plates reinforced by one to fourteen stiffeners in two orthogonal directions.

2. Governing differential equations

2.1. The principle of minimum potential energy

The principle of minimum potential energy can be derived from the principle of virtual work for conservative systems. If a system is in a position of stable equilibrium, its total energy is a minimum [2]. The total potential energy, Π, of a system consists of two parts which can be expressed as

$$\Pi = U + V \tag{1}$$

where U is the internal strain energy in the system and V is the potential energy of external loads. When an elastic body is in equilibrium, the first variation of the total potential energy has a stationary value, that is:

$$\delta\Pi = \delta(U + V) = 0 \tag{2}$$

Expressions for the total potential energy of a stiffened plate system are first derived in this section. The calculus of variations techniques are applied to obtain the differential equations of equilibrium and the associated boundary conditions.

2.2. Total potential energy of eccentrically stiffened plates

Consider a eccentrically stiffened plate system with length of a in x-direction and width of b in y-direction as shown in Fig. 1. Only the i-th stiffener in the x-direction and j-th stiffener in the y-direction are illustrated. The stiffeners in x- and y-direction are designated as x-stiffeners and y-stiffeners, respectively. The plate is assumed to be made with orthotropic material. Stiffeners are defined as elastic homogeneous and isotropic but the material properties may be different between x-stiffeners and y-stiffeners. Partial composite action is assumed along the plate-stiffener interface where small slippage may occur. The classical small deflection theory of thin plates is assumed which indicates the deflection in the z-direction, w, is a function of x and y only, and the deflection is small relative to the plate thickness. Plane sections are assumed to remain plane during deformation and deflections due to shear may be neglected. The normal strain and stress distributions can be determined from the conventional linear elastic, small deformation theory for beams and plates. Fig. 2 shows the strain distribution of a composite T-beam in which the plate and the stiffener act as the flange and web, respectively.

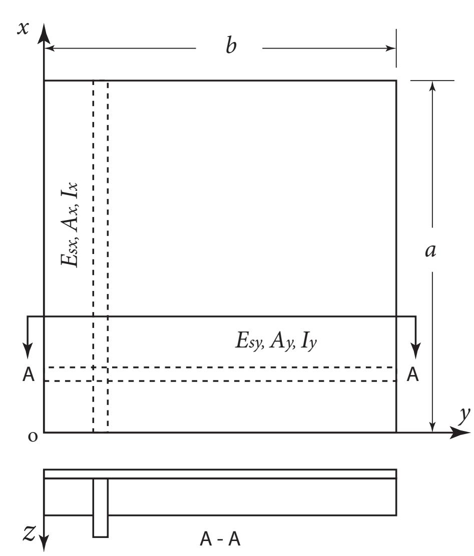

Fig. 1. Eccentrically stiffened plates.